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Article

Social Thermodynamics, Social Hydrodynamics and Some Mathematical Applications in Social Sciences

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Abstract: Based on the sameness for men or any elements in the social systems, we search the social thermodynamics, and possible entropy decrease in social sciences. Next, using the similar formulas of the preference relation and the utility function, we propose the confidence relations and the corresponding influence functions that represent various interacting strengths of different families, cliques and systems of organization. This produces a multiply connected topological economics. Further, we discuss the binary periods of the political economy by the complex function and the elliptic functions. Using the nonlinear equations of hydrodynamics we may research the formulations of the binary and multiple centers in various social systems. Finally, we propose the developed directions of society and a unification of simplicity and complexity from a new tree-field representation. The application of mathematics and physics will be an important direction of modern social science at 21 century.

Keywords: social science, mathematics, thermodynamics, entropy, nonlinearity, hydrodynamics, topology, complex function

1. Introduction

It is long time ago that mathematics is applied in various social sciences. This may trace to its source: Chinese Yi (I Ching, or The Book of Change) and Pythagoras. Both all assume that the odd numbers correspond to yang, and the even numbers correspond to yin. Pythagoras proposed that everything is number. In Yi all of world are composed of yin (- -, R) and yang (—, L). In modern

physics any object is composed of quarks and some smaller subquarks. The simplest subquark model is the rishon (or quip) model (Harari, 1979; Shupe, 1979), whose elements T, V (or a^+, a^0) correspond to R and L, so first generation quark-lepton is e^+ = TTT, ν_e =VVV and u=(TTV,TVT,VTT), $\bar{d}$ =(TVV,VTV,VVT). This is the same with the Eight Diagrams composed of R and L (Chang, 1989; 2009).

Coleman published a book entitled "Introduction to Mathematical Sociology" in 1964. Its basic is that the development of any science depends on systematic empirical study, and the study of human society can become a true science. Author examined the contribution that various mathematical techniques might make to systematic conceptual elaboration of social behavior. These mathematical tools include simple algebra, matrix analysis, calculus, stochastic method and probability theory, etc. Fararo discussed a non-technical essay as the nature of mathematical sociology (Fararo, 1969) and the reflections on mathematical sociology (Fararo, 1997). "Journal of Mathematical Sociology" launch its first issue in 1971. Sorensen researched the mathematical models in sociology (Sorensen, 1978). Edling searched mathematics in sociology (Edling, 2002).

The mathematical sociology applied probability, linear programming, cybernetics, game theory, operational research, optimum scheme theory, etc. Further, the mathematical social science includes the mathematical sociology, the mathematical economics, the social metrology, the metrological historiography, the econometrics, the economic metrology, the polimetrics, geopolitics, the social biology, the disaster sociology, the disease sociology, mathematical history, etc. They may apply analysis, geometry, topology and so on.

Based on synergetics Haken investigated the social public opinion, employment and unemployment, the business model, etc. (Haken, 1977; 1983). Further, Haken, et al., applied theories, concepts and methods in synergetics, and found the quantitative sociology (Weidlich, et al. 1983). They suppose the existence of various social fields on politics, economy, society, culture, religion and so on, and proposed corresponding quantitative models. By using a following equation

$$\frac{dX}{dt} = I + RX - CX^3, \tag{1}$$

They discussed quantitatively employment and unemployment, etc. Prigogine, et al. (1984), investigated quantitatively the evolution of city, the market model and the communications and transportation, etc.

Based on the synergetics, we proposed the social synergetics and the four basic theorems, in which theorem of perfect correlation on humanity is researched mathematically. Otherwise, we proposed an equation of corruption, and discussed quantitatively some threshold values for a social system into corruption. From synergetics we derived the Lorenz model, which may be a visualized

two-party mechanism as a type of stable structure in democracy. A developed direction of society should be the combination from macroscopic to microscopic order, from an actual capable handling to an ideal pursuance (Chang, 2013a). Further, we proposed the nonlinear whole sociology and its four basic laws, and the nonlinear theory of economic growth (Chang, 2013b).

2. Social Field and Social Thermodynamics

Society forms various social fields, which may be the scalar, vector, spinor, tensor fields, etc. These fields must be nonequilibrium and nonuniform, from this forms gradient for the scalar field, and divergence and rotation for the vector field. In the n-dimensional space, the gradient is:

$$Gradu = \sum_{i=1}^{n} \frac{\partial u}{\partial x_i} \vec{i}_i . \tag{2}$$

The n-dimensional divergence is:

$$Div\vec{P} = \sum_{i=1}^{n} \frac{\partial P_i}{\partial x_i} . \tag{3}$$

The n-dimensional rotation is more complex, and is probably:

$$\text{RotP} = \sum_{i,j=1}^{n} \left(\frac{\partial P_j}{\partial x_i} - \frac{\partial P_i}{\partial x_j} \right) . \tag{4}$$

Their flow or movement may be described by the diffusion equation and the equations of hydrodynamics, etc. Helbing developed a mathematical model for behavioral changes under the influence of a social field (Helbing, 1994).

We assume that they first may be simplified and unified to an extensive temperature field, which is the simplest scalar field. From this the social thermodynamics can be obtained. The social thermodynamics is a part of the social physics (Lepkowski, 1979; Chang, 2013b). Its basis is that many-particle systems with some macrostates can be found everywhere in nature (Greiner, et al. 1995), and men or any elements in the social systems are approximately the same much like atoms. From this we searched the social thermodynamics and the five fundamental laws of social complex systems, which are analogue with thermodynamics and statistics. We researched different relations among social elements, and the moderate degree on the entropy production in systems and on the input negative entropy flow for open systems (Chang, 2013b).

The difference of the extensive temperature forms the force, which is namely a vector field. Force derives movement, and forms the social hydrodynamics. Force and movement obtain the social potential energy U and the social kinetic energy K, and we discussed generally the four variables and the eight aspects in social physics (Chang, 2013b).

The social deprivation, in particular, the relative deprivation (Runciman, 1966; Chakravarty, et al., 1997), is also origin of a difference of the extensive temperature. The basis of the social public opinion should be the social thermodynamics and the social hydrodynamics. It includes the social combustion theory (Niu, 2001; 2009), the social shock wave theory and the entropy of social action theory (Niu, 2009).

Since a state of single element in any complex social and natural systems is indeterminate and fluctuated, we discussed the social thermodynamics that analyze mainly the total system, which obeys the statistical rules, and may apply some methods of thermodynamics and statistics, since many completely different systems obey the common and very general concepts of thermodynamics, which are a great extent independent of special models (Greiner, et al. 1995).

Because the basis of thermodynamics is the statistics, in which a basic principle is statistical independence: The state of one subsystem does not affect the probabilities of various states of the other subsystems, since different subsystems may be regarded as weakly interacting (Landau, et al. 1980). It shows that various interactions among these subsystems should not be considered. But, if various internal complex mechanism and interactions cannot be neglected, a state with smaller entropy (for example, self-organized structure) will be able to appear under some conditions. In this case, the statistics and the second law of thermodynamics should be different (Chang, 1994, 1997, 2005, 2012a, 2013c). Because internal interactions bring about inapplicability of the statistical independence, entropy decrease in an isolated system is possible, in particular, for attractive process, internal energy, system entropy and so on. Therefore, we proposed that a necessary condition of entropy decrease in isolated system is existence of internal interactions. The internal interactions often are related with nonlinearity (Chang, 1994, 1997). For any isolated system we proposed a generalized formula (Chang, 2005):

$$dS = dS^a + dS^i , \tag{5}$$

where dS^a is an additive part of entropy and is always positive, and dS^i is an interacting part of entropy and can be positive or negative. Eq.(5) is similar to a well-known formula:

$$dS = d_i S + d_e S , \tag{6}$$

in the theory of dissipative structure proposed by Prigogine. But, two formulae are applicable for internal or external interactions, respectively. Based on the Eq.(5), a sufficient and necessary condition of entropy decrease in isolated system is expressed quantitatively (Chang, 2012a):

$$0 > dS^i > -dS^a , \text{ i.e., } |dS^i| > dS^a \text{ (for negative } dS^i\text{).} \tag{7}$$

In usual cases, the condition corresponds to that in isolated systems there are some stronger internal attractive interactions. The analysis for many experiments and theories shows that the second law of the thermodynamics should be developed.

Human possesses a special duality unifying Bose-Einstein statistics and Fermi-Dirac statistics, i.e., unifying inclusion and exclusion. In social sciences both are respectively knowledge, love, internationalism, etc., and right, consort, nation, domain, nationalism, etc. They are two different relations, which correspond to that we proposed the fourth exclusion or inclusion law distinguished strictly for objects in the social complex systems (Chang, 2013b). Their coordination is namely the unification at high quality. For instance, FD statistics may be applied to arrange order and rule like the Confucianism and democracy.

We research possible entropy decrease in some social systems. The certain commodity and the two parties of business construct an isolated system in economics, but it will not usually tend to a disorder state, a price will reach stable through some arrange (i.e., internal interaction), and the system will become order. It is well known that if some people exchange views (i.e., internal interaction in isolated system), new thought and information will be produced. In some social systems, even the constitution and circumstance are the same, so long as the structure is optimized, then the system will become more order. In a word, the evolution of Earth and biology, and human history are always not a disorder process. Of course, human aftertime is also not a nice heaven without misery. Human existence and development, order or disorder, determine on the human internal relations, and on the interactions between humanity and other environment.

In fact, any stable objects and their formations from particles to stars are accompanied with internal interactions inside these objects. Various evolutionary processes from astronomy to biology and social systems cannot be only increase of entropy. We researched some possible tests for entropy decrease in isolated systems in physics, chemistry, biology and so on (Chang, 1994, 1997, 2005, 2012a, 2013c). It should be confirmed by many stable states in Nature. At present entropy increase as a new world view (Rifkin, et al. 1981) already became a conviction and a scientific alienation.

3. Social Topology and Multiply Connected Topological Economics

If a social field is continuous, it will connect with topology, and forms the topological sociology. If its continuation is broken due to some force, etc., it will be able to be described by the catastrophe theory (Thom, 1983).

From the synergetic equations, we obtained the equations on the rule of law, and may prove mathematically that a society of the rule of law cannot lack any aspect for three types of the legislation, the administration and the judicature (Chang, 2013a). This corresponds to Borromean rings with three loops (Fig. 1) in topology (Adams, et al. 2008; Chichak, et al. 2004; Mao, et al. 1997; Periwal, 1993; Wikoff, et al. 2000), here united they stand, divided they fall. It is also a general model with three elements.

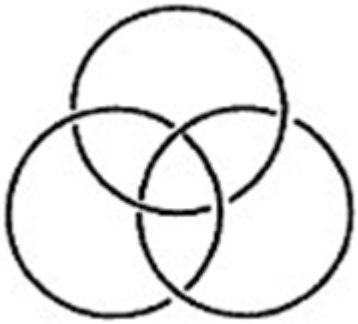

Fig. 1: Borromean rings

The economic systems are some particular social systems, and should be whole and nonlinear. Grandmont (1986) and Puu (1989) published *Nonlinear Economic Dynamics*. Peters (1991) studied chaos and order in the capital markets from a new view of cycle, prices and market volatility.

The topological economics is known. In mathematical economics the fixed-point theorems of topology are used to prove the Nash equilibrium for n-person games. Arrow and Debreu presented a general model of Walrasian equilibrium theory, and proved the existence theorem of equilibrium for a competitive economy by topology (Arrow, et al. 1954). Then McKenzie (1959), Debreu (1962), et al., developed the competitive equilibrium theory. Differential topology is introduced into economics, and Debreu (1976) discussed two detailed questions.

In the microeconomic theory of consumer behavior either a utility function or a binary relation can describe the preferences of an individual. The strict equivalence of these two primitive concepts, ordinal utility functions and preference relations, was first axiomatized by Debreu (1959; 1964). He studied the concept of cardinal utility in three different situations by means of the same mathematical result that gives a topological characterization of three families of parallel straight lines in a plane (Debreu, 1959), and discussed that for every continuous complete and transitive binary relation $\geq$ defined on an arbitrary subset X of the commodity space R, there is a continuous utility representation; that is, there is a continuous function u of X into R such that $u(y) \geq u(x)$ if and only if $y \geq x$. Therefore, the more basic concept of preferences is applied instead of utility by means of a topology or a metric on the space of preferences. Undoubtedly, it is a great contribution for economics.

In microeconomics we introduce the confidence relations that represent various interacting strengths of different families, cliques and systems of organization. It is an important human relation in economics, even is independent of economic results. The confidence relation can be defined by a similar method with the preference relation in consumer theory (Chang, 2010, 2012b).

The confidence relation $\geq$ defined on the choice set X is a complete preordering, continuous and strictly monotone. This requires (Fuente, 2000):

(1). Reflexivity: $\forall x \in X, x \geq x$;

(2). Completeness: $\forall x, y \in X$, either $x \geq y$ or $y \geq x$ or both;

(3). Transitivity: $\forall x, y, z \in X$, [$x \geq y$ and $y \geq x$] $\Rightarrow x \geq z$.

Then $\geq$ can be represented by a real-valued, continuous and increasing payoff function.

Further, the definition of the influence function I is similar with the utility function: A real-valued function $I^i : X^i \to R$ represents a confidence preordering $\{\geq_i\}$ defined on the choice set X^i of agent i if $\forall x, y \in X^i, x \geq_i y \Leftrightarrow I^i(x) \geq I^i(y)$. The influence function that represents a confidence preorder is not uniquely defined. Any monotonically increasing transformation $\varphi(\ )$ of I( ) will represent exactly the same confidences, because with $\varphi(\ )$ strictly increasing, we have

$$I(x) \geq I(y) \text{ if and only if } \varphi[I(x)] \geq \varphi[I(y)], \tag{8}$$

for all $\forall x, y \in X$. Hence I( ) is an ordinal influence function. The sign of the difference $I(x) - I(y)$ is important because it tells us which outcome is confided, but the value of this difference is meaningless, as it will change with any nontrivial increasing transformation $\varphi(\ )$. It is also a basic characteristic of topology, where those concrete spacing values are meaningless. Although the influence function is similar to the utility function that obeys the law of diminishing marginal utility, but the influence function seems to obey the law of augmenting lust for power.

The confidence relation, the corresponding influence function I( ) and the function $\varphi(\ )$ can affect products Q, profit and prices, etc., in an economic system. But, they are usually independent of economic results, and sometimes are stochastic, even change suddenly. In a continuous topological manifold of economics they break easily original structure, and form a new hole or branch region. This will construct a multiply connected topological manifold. In an image the economic structure is a cup, while the influence function is a handle.

In a multiply connected region of topology there is a famous Euler-Poincare formula

$$\sum_{m=1}^{n} (-1)^m a_m = \sum_{m=1}^{n} (-1)^m p_m \, . \tag{9}$$

For a convex polyhedron, a_0, a_1, a_2 denote the number of vertices, edges, and faces, respectively; p_m is the mth Betti number of complex K. This may be considered intuitively as the numbers of m-dimensional holes in K, or is the number of (m+1)-dimensional chains that must be added to K so that every free m-cycle on K is a boundary (Hocking, et al. 1961). The number $\sum_{m=1}^{n} (-1)^m a_m$ is called the Euler characteristic of the complex K. In the polyhedron $p_0 = p_2 = 1, p_1 = 2p$, p is the deficiency of a curved surface. In 2-dimensional curved surface, $a_0 = a_1 + 1 - 2p$. Assume that vertices represent the number of market, which is direct proportional to the sales volume *y* and the

profit, and edges represent the market network. But the multiply connected economy brings the profit decrease. In this case there is a defective profit due to the deficiency *p*.

In some systems of organization the profit maximization and the confidence relations are inseparable. The aim of a pure producer is the profit maximization

$$\pi(y,w)=\max_{x,y}\{py-wx\}, \tag{10}$$

where *y* and *x* are output and input, *p* and *w* are output and input prices. For a social system with the influence function I, we should define an aim function as

$$A=\pi+I. \tag{11}$$

When the changes of the product and the influence are independent one another, they may be a node or saddle point (Chang, 2010, 2012b). Various powers produce usually the economic wormhole and various corruptions. It is a mathematical application to economics. The form of the influence function can be an unrestricted function, even a stochastic function. Perfect competition prevails that each producer and consumer regards the prices paid and received as independent of his own choices (Arrow, et al. 1954). An economy with the confidence relations and the influence functions is a type of imperfect competitive economic systems, and break the symmetries in economic topology. They are not homeomorphic spaces. Usually this structure will hinder the economic development. If the confidence relations and the influence functions have *p*-levels or *p*-types, i.e., $\sum_{i=1}^{p} I_i(Q)$, they will construct a multiply connected normal curved surface with the deficiency *p*. When the influence function large enough achieves a certain threshold value, the economic elasticity of topological structure will be broken, and a new hole will appear. Unified market economy will be riddled with holes. This will form a new multiply connected topological manifold. As an example, using the concept of general relativity a large influence as mass of general relativity forms a pit in the economic system. A very strong pit can construct a wormhole, sometimes called the Einstein-Rosen bridge. Therefore, some capital will pass through a throat into another topological space (Fig.2), or from a region to another region in the same space. This model will may describe a loss of capital (including waste, and corruption).

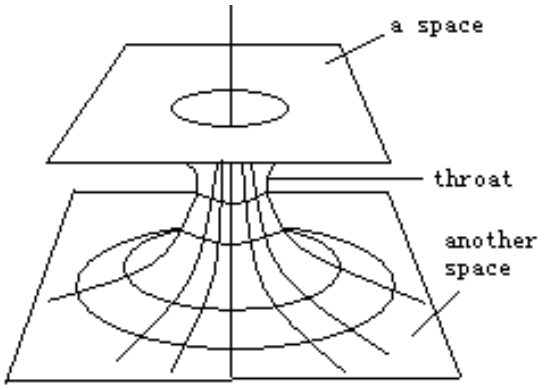

Fig.2: The wormhole model in social economics from a space into another topological space

4. Binary Political Economy and Complex Functions

A complex world may be described by the complex functions, which obtain often multivalue. For instance, a function of single value e^{nx} is a logarithmic spiral curve, and corresponds to an exponential economic growth. It develops to a complex function:

$$e^{inx} = \cos nx + i \sin nx . \tag{12}$$

This is a multivalue function, and corresponds to the periodic economic growth. A complex equation $p(\alpha, y) = 0$ is n-th order of y, for a plane of $x = \alpha$, there are the n-layers curved surfaces of y. In quantum mechanics the imaginary numbers are introduced, and the multivalues and many worlds are obtained. This corresponds to the variety of world, and R.Cotes in 1714 called "harmonic measurement" (Stillwell, 2001).

If the political economy is an economy chaperoned polity, it will produce consequentially a binary economy. The political economy is usually imperfect economic question, even completely is not an economic question for some particular cases. It is not a strict economic rule, because in this case economy is only an appendage of polity. The economy will change along with polity.

The political economy as a multiply connected topological economics can be described by the complex function and the elliptic functions, which have two periods of economy and policy. The complex function corresponds to a complex social system of economy and polity. This is a nonlinear whole economy. In the multiply connected topological economy, economy corresponds to a real part, and policy and relation, etc., correspond to an imaginary part. A complex plane corresponds to the surface of the Riemann sphere, which is called a stereographic projection (Brown, et al. 2009).

The Cauchy-Goursat theorem (Brown, et al. 2009) is the closed contour to simply connected domains

$$\oint_C f(z)dz = 0 . \tag{13}$$

For multiply connected domains

$$\oint_C f(z)dz = \sum_{k=1}^{z} \oint_{C_k} f(z)dz . \tag{14}$$

An elliptic function is a meromorphic function on the complex plane (Lang, 1987), and has the double periods:

$$f(z + m\omega + n\omega') = f(z) . \tag{15}$$

Here ω and ω' are the two different basic periods. In this case, the economical development will possess two periods of economy and policy, respectively. It includes the Weierstrass elliptic function:

$$W(z) = \frac{1}{z^2} + \sum_{\omega \in L} [\frac{1}{(z-\omega)^2} - \frac{1}{\omega^2}], \tag{16}$$

where the sum is taken over the set of all non-zero periods, denoted by L (Lang, 1987). Its expansion shows different influence degrees of economy or policy, for example, the confidence relation for different families, cliques and systems of organization (Chang, 2012b). There is a theorem (Lang, 1987): Assume that the elliptic function *f* has no poles on its boundary, then the sum of the residues of *f* is 0, i.e.,

$$2\pi i \sum \mathrm{Re}\, sf = \int_{\partial P} f(z)dz = 0 . \tag{17}$$

It may describe a special case: Economy and policy cancel out, so economy cannot be developed. Its corollary is: An elliptic function has at least two poles on the torus, i.e., the two centers of economy and policy exist simultaneously. The elliptic curves with singular invariants have complex multiplication from an imaginary quadratic field.

The Weierstrass zeta function is (Lang, 1987):

$$\varsigma(z, L) = \varsigma(z) = \frac{1}{z} + \sum_{\omega \in L} [\frac{1}{z-\omega} + \frac{1}{\omega} + \frac{z}{\omega^2}] . \tag{18}$$

There have $\varsigma' = -W(z)$ and

$$\varsigma(\lambda z, \lambda L) = \frac{1}{\lambda}\varsigma(z, L) . \tag{19}$$

In this case it is a scaling invariance. There is a constant $\eta(\omega)$ such that

$$\varsigma(z + \omega) = \varsigma(z) + \eta(\omega) . \tag{20}$$

Their separation is namely a connected graph becomes the non-connected graphs (Diestel, 2000; Bollobas, 2002) in graph theory of the political economy.

In Chinese traditional society the periodic changes of dynasty shown the two periods of economy and policy, and both influence each other. The two periods of economy and corruptions are also influenced each other.

When $\alpha \in M_2^+(Z)$ (the sets of 2×2 matrix with components in Z) and $\det \alpha = N$, then $j \circ \alpha$ is a modular function of level N (Lang, 1987). Here α is matrix, so it may correspond to the input-output model, and two aspects of economy and policy. It may have higher levels, which include economy and policy. In the elliptic function there are the subfields of the modular function field. In the multiply connected topological economy, economy and policy are different subfields.

The multiply connected topological economy may be extended to various relations between economy and other politics, family, religion, etc. Further, it may be developed to many regions of without direct relations with economy, for example, welfare, environment, and full employment, etc.

5. Social Hydrodynamics and Formulations of Binary and Multiple Centers

Since various social forces and potentials exist universally, the corresponding movements and fluids form the social hydrodynamics. A classic example is the population migration. Its basis is a human nature on tend to advantages and avoid disadvantages. Further, this may include human behaviors restricted by the causality. The rule of law and the moral standard in society all are various fluid paths. It may be related with the criminal economics proposed by Gary S.Becker.

The binary stars are very common in astronomy. Based on the basic equations of the hydrodynamics, we obtained a nonlinear dynamical model of formation of binary stars by the qualitative analysis theory of nonlinear equation (Chang, 2000, 2007). Under different conditions many singular points or a single central point in an evolutional process corresponds to the multiple stars or a single star. The nonlinear interaction plays a crucial role, and is necessary condition of the formation of the multiple stars. This method and model may be extended and developed. From this the social hydrodynamics may describe the formulations of the binary and multiple centers on public opinion, party (Chang, 2013a), market, city and so on.

The nonlinear hydrodynamic equations are (Chang, 2007):

$$\rho \frac{dV}{dt} = \rho[\frac{\partial V}{\partial t} + (V\nabla)V] = F + \frac{e}{c}V \times B - gradp + \mu \Delta V + \frac{\eta}{3} graddivV . \tag{21}$$

In the two-dimensional plane, the equations become

$$\frac{\partial \rho u}{\partial t} = -(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y})\rho u + F_x + \frac{e}{c}B_z v - \frac{\partial p}{\partial x} + \mu(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2})u + \frac{\eta}{3}\frac{\partial}{\partial x}(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}), \tag{22}$$

$$\frac{\partial \rho v}{\partial t} = -(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y})\rho v + F_y - \frac{e}{c}B_z u - \frac{\partial p}{\partial y} + \mu(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2})v + \frac{\eta}{3}\frac{\partial}{\partial y}(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}). \tag{24}$$

Here μ and η correspond to some resistances in a social field. For the simplified equations, its characteristic equation is

$$\lambda^2 - T\lambda + D = 0 . \quad (25)$$

A plane system has usually three singular points. For general cases: 1) When D<0, a corresponding singular point is a saddle point. 2) When D>0, the point is a focal point if Δ<0, or a nodal point if Δ>0. 3) When T<0, the point is a stable sink. For T>0, the point is an unstable source.

It is common that their energies and masses are bigger, so are the attractions, and the formulations of the centers are also easier. Both evolutions of nebula and society are also similar. The formulations of city pass through fluctuations attach order, and have Christaller model (Prigogine; et al. 1984).

Further, the social tensor fields may be described by the general relativity, which is also the basis of the most exact evolutionary theory of large scale structures. We calculated the 2+1 dimensional plane equations of gravitational field, and based on these equations we discussed the evolutions of disk nebula by the qualitative analysis theory, in which the multiple stars or single star are formed for different conditions. This is the most exact model of formation of binary and multiple stars (Chang, 2013d).

The mathematical applications may generally extend to the nonlinear social sciences, which include chaos, fractal and soliton theories. For instance, chaos corresponds to the economic crisis and war burst, etc. The formations of city and public opinion, and various managements possess all the fractal structures. “Dark horse” in society may be described by soliton. Moreover, we may discuss and propose various crossing theories combined social sciences with other sciences, for example, new possible the chemical sociology, astronomy and sociology, etc.

6. Discussion

We extended the tree in the graph theory to a new tree-field representation (Fig.3). It includes two parts: tree (a,b,d,s) and field (C,S). A field is a set of legion small trees. They can transform each other between tree and field. This is a unification of simplicity (tree) and complexity (field), and may be applied to various complex systems on science, politics, economy, philosophy and so on (Chang, 2012c). Further, it may be extended to the whole graph theory G=(V,E,F), here F is a set of small graphs.

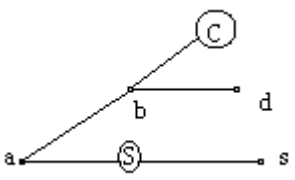

Fig. 3: The tree-field representation

For the developments of science, tree represents principal science, and field corresponds to secondary science, which includes pre-science and potential science, etc. Tree is the paradigm in Kuhn's science development. For synergetics tree is an order parameter and field is the general parameter. In economy the tree represents the main aspect of the economic growth for a time, and the field represents other economic aspects, but which produce probably new direction of economic growth (Chang, 2012c), for example, the solar energy, the wind energy and so on in new energy sources. For the political economics as a multiply connected topological economics, the politics is often first, and corresponds to tree, and economy is attached, and corresponds to field. In a word, the tree-field representation may be applied to many aspects, in particular, for analysis of various complex systems.

The economic growth depends on new developments of science and technology. They include from the fishing-hunting society and the agricultural society to the industrial society, from the steam engine and electric motor to information. Some concrete auto industry and real estate, etc., have all the developing periods. The developed directions of China and world should be the new higher technologies. The transformation from energy to quality on social development (Chang, 2000) will be a main direction of society, which includes the safe food, the health protection work on body and mind, etc.

In a word, the applications of mathematics and physics in social sciences have enormous potentiality, and will be an important developed direction of modern social science at 21 century.

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