

Multivariate Calibration Estimation for Domain in Stratified Random Sampling

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Abstract: In sample surveys that incorporate auxiliary information, the precision of the survey estimates is always improved when multiple auxiliary information are available. Calibration is used in survey sampling to include auxiliary information. In the presence of powerful auxiliary variables, the calibration estimation meets the objective of reducing both the non-response bias and the sampling error. In this paper, multivariate calibration estimator for domain totals in stratified random sampling design is proposed using multiple auxiliary variables. Analytical approach for obtaining optimum calibration weights is developed. The efficiency gain of the proposed calibration based approach estimator vis-à-vis conventional estimators is studied through simulation.

Keywords: auxiliary information, domain estimation, multivariate calibration estimation, optimum calibration weights, stratified sampling

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1. Introduction

The term calibration estimation was introduced by Deville and Sarndal (1992). They considered the problem of estimation of finite population total through the calibration approach. A theoretical

framework of the approach was given. Thus, expression for estimators, variance of the estimators and variance estimators were given. Different distance functions were considered to minimize the distance between the original weights and the new weights.

Calibration estimation adjusts the original design weights to incorporate the known population totals of auxiliary variables. The calibration weights are chosen to minimize a given distance measure (or loss function) and these weights satisfy the constraints related auxiliary variable information.

In survey sampling many authors such as Wu and Sitter (2001), Montanari and Ranalli (2005), Farrel and Singh (2005), Arnab and Singh (2005), Estavao and Sarndal (2006), Kott (2006), Singh (2006a, 2006b), Sarndal (2007), Kim and Park (2010), defined some calibration estimators using different constraints.

Sarndal and Lundstrom (2005), Ardilly (2006) and Kott (2006) used calibration approach to achieve non-response adjustment for population parameters and Clement et al (2014a) used calibration approach to adjust for non-response in domain estimation.

In stratified random sampling, calibration approach is used to obtain optimum strata weights. Kim, Sungur and Heo (2007), Koyuncu and Kadilar (2013) defined some calibration estimators in stratified random sampling for population characteristics and Clement et al (2014b) defined calibration estimators for domain totals in stratified random sampling. Rao et al (2012) proposed multivariate calibration estimator for population mean in stratified random sampling when information on two auxiliary variables are available.

In this study, under the stratified random sampling scheme, multivariate calibration estimator is proposed for domain totals when information on three auxiliary variables is available.

2. Methodology

2.2. The reviews of Calibration Estimation

Consider a finite population U of N elements

$$U = (U_1, U_2, \dots, U_N) \quad (1)$$

The estimator of the population total Y from a simple random sample of size n taken without replacement is given by $\hat{Y} = \frac{N}{n} \sum_{i=1}^n y_i$. Under a probability sampling design P , with probability $P(s)$, Deville and Sarndal (1992) gave an unbiased estimator of the population total as $\hat{Y} = \sum_{i=1}^n d_i y_i$; where $d_i = \frac{N}{n}$ is the design weight associated with unit i and defined as the inverse of the inclusion probability π_i ($\pi_i > 0$ for each i), where $\pi_i = \sum_{s \in S} P(s)$

When “bad” sample is observed; Deville and Sarndal proposed the used of an auxiliary variable x_i with calibration weights w_i to adjust for the “bad” sample, where the population total for the auxiliary

variable was defined as $t_x = \sum_{i=1}^N x_i$. Under an ideal condition, the estimator of the population total of the auxiliary variable should be $\hat{t}_x = \sum_{i \in S} d_i x_i$; but because of the presence of the “bad” sample, they observed that $\hat{t}_x = \sum_{i \in S} d_i x_i$ was far from $t_x = \sum_{i=1}^N x_i$.

To adjust for the “bad” sample they chose weights w_i for $i \in s$ (called the calibration weights) such that the w_i 's are close to the d_i 's and then assumed an unbiased estimator of the population total of the auxiliary variable to be $\hat{Y}_x = \sum_{i \in S} w_i x_i$

The new estimate of the population totals after the adjustment is $\hat{Y}_w = \sum_{i \in S} w_i y_i$. Following the chi-square distance measure of the form, $\sum_{i \in S} \frac{(w_i - d_i)^2}{d_i q_i}$ where q_i is a tuning parameter and subject to the calibration constraint, $\hat{Y}_x = \sum_{i \in S} w_i x_i = \sum_{i \in S} x_i$, the corresponding Lagrange Multipliers is:

$$L = \sum_{i \in S} \frac{(w_i - d_i)^2}{d_i q_i} - 2\lambda \sum_{i \in S} w_i x_i \quad (2)$$

Differentiating (2) with respect to w_i and setting it to zero gives the calibration weights as

$w_i = d_i + \lambda d_i q_i x_i$ where $\lambda = \frac{\sum_{i \in S} w_i x_i - \sum_{i \in S} d_i x_i}{\sum_{i \in S} d_i q_i x_i^2}$. The resulting calibration estimator is

$$\hat{Y}_W = \sum_{i \in S} w_i y_i = \hat{Y} + (\hat{Y}_x - \hat{t}_x) \hat{\beta}, \quad \text{where } \hat{Y} = \sum_{i=1}^n d_i y_i, \hat{Y}_x = \sum_{i \in S} w_i x_i, \hat{t}_x = \sum_{i \in S} d_i x_i \text{ and } \hat{\beta} = \frac{\sum_{i \in S} d_i q_i x_i y_i}{\sum_{i \in S} d_i q_i x_i^2}.$$

2.3. Multivariate Calibration Estimation

Consider the finite population of equation (1) divided into D domains; U_1, U_2, \dots, U_D of sizes N_1, N_2, \dots, N_D respectively. Domain membership of any population unit is unknown before sampling. It is assumed that domains are quite large.

Consider a stratified random sampling design with H strata and such that n_h elements are considered from N_h in stratum h , $h = 1, 2, \dots, H$. It is assumed that domain coincide with stratum. Then, the design weights are $d_j = N_h/n_h$ for all j in stratum h , $j = 1, 2, \dots, N_h$.

Let y_{hj} be the y value of the j th element in stratum h , $h = 1, 2, \dots, H$ and $j = 1, 2, \dots, N_h$

Let x_{hj} be the x value of the j th element in stratum h , $h = 1, 2, \dots, H$ and $j = 1, 2, \dots, N_h$ where y and x are the study variable and auxiliary variable respectively.

Following from Gamrot (2006), for a typical d th domain U_d the domain total is:

$$Y_h = \sum_{j=1}^{N_h} y_{hj}; \quad h = 1, 2, \dots, H. \quad (3)$$

Let the conventional estimator of the domain totals under stratified random sampling design be given by

$$y_{st} = \sum_{h=1}^H w_h y_{hj} \quad (4)$$

where w_h is the design weights and is given by $w_h = N_h/N$.

In the presence of multiple auxiliary variables, a multivariate calibration estimator of the domain totals under stratified sampling is given by

$$y_{st}^* = \sum_{h=1}^H w_h^* y_{hj} \quad (5)$$

where w_h^* is the new weights called the optimum calibration weights.

Let X_i , $i = 1, 2, \dots, m$ be the available auxiliary variables. Then the following calibration constraints are defined:

$$\sum_{h=1}^H w_h^* x_{hij} = X_i \quad ; \quad i = 1, 2, \dots, m \quad (6)$$

Let the loss function L be defined as:

$$L = \sum_{i=1}^m \sum_{h=1}^H \frac{w_h}{q_{hi}} \left(\frac{w_h^*}{w_h} - 1 \right)^2 \quad (7)$$

2.3.1. Derivation of optimum calibration weights for multivariate calibration estimation

According to Rao et al (2012), the problem of determining the optimum calibration weights w_h^* may be formulated as a Mathematical Programming Problem (MPP) as follows:

Minimize

$$Z = \sum_{h=1}^H \frac{w_h}{q_h} \left(\frac{w_h^*}{w_h} - 1 \right)^2 \quad (8)$$

Subject to

$$\begin{aligned} \sum_{h=1}^H w_h^* x_{hj1} &= X_1 \\ \sum_{h=1}^H w_h^* x_{hj2} &= X_2 \\ &\vdots \\ \sum_{h=1}^H w_h^* x_{hjm} &= X_m \end{aligned} \quad (9)$$

where $q_h = \sum_{i=1}^m q_{hi}$ and

$$w_h^* \geq 0; h = 1, 2, \dots, H \quad (10)$$

The corresponding Lagrange's multipliers are:

$$L(w_h^*, \lambda_i) = \sum_{h=1}^H \frac{w_h}{q_h} \left(\frac{w_h^*}{w_h} - 1 \right)^2 - 2 \sum_{i=1}^m \lambda_i \left(\sum_{h=1}^H w_h^* x_{hij} - X_i \right) \quad (11)$$

The necessary conditions for the solution of optimum calibration weights are:

$$\frac{\partial L}{\partial w_h^*} = \frac{\partial L}{\partial \lambda_i} = 0 \quad (12)$$

That is:

Differentiating (11) with respect to w_h^* and equating to zero gives:

$$w_h^* = w_h \left(1 + q_h \sum_{i=1}^m \lambda_i x_{hij} \right) \quad (13)$$

and differentiating (11) with respect to λ_i and equating to zero gives:

$$\begin{aligned} \frac{\partial L}{\partial \lambda_1} &= -2 \left(\sum_{h=1}^H w_h^* x_{hj1} - X_1 \right) = 0 \Rightarrow \sum_{h=1}^H w_h^* x_{hj1} = X_1 \\ \frac{\partial L}{\partial \lambda_2} &= -2 \left(\sum_{h=1}^H w_h^* x_{hj2} - X_2 \right) = 0 \Rightarrow \sum_{h=1}^H w_h^* x_{hj2} = X_2 \\ &\vdots \\ \frac{\partial L}{\partial \lambda_m} &= -2 \left(\sum_{h=1}^H w_h^* x_{hjm} - X_m \right) = 0 \Rightarrow \sum_{h=1}^H w_h^* x_{hjm} = X_m \end{aligned} \quad (14)$$

On substituting (13) into (14) gives:

$$\begin{aligned} \sum_{h=1}^H w_h \left(1 + q_h \sum_{i=1}^m \lambda_i x_{hij} \right) x_{hj1} &= X_1 \\ \sum_{h=1}^H w_h \left(1 + q_h \sum_{i=1}^m \lambda_i x_{hij} \right) x_{hj2} &= X_2 \\ &\vdots \\ \sum_{h=1}^H w_h \left(1 + q_h \sum_{i=1}^m \lambda_i x_{hij} \right) x_{hjm} &= X_m \end{aligned} \quad (15)$$

Solving the system of equations in (15) for λ_i and on substituting the values of λ_i into (13), the optimum calibration weights are obtained as:

$$w_h^* = w_h + w_h q_h (\lambda_1 x_{hj1} + \lambda_2 x_{hj2} + \cdots + \lambda_m x_{hjm}) \quad (16)$$

Substituting (16) into equation (5); a multivariate calibration estimator of domain totals under stratified random sampling is obtained as;

$$y_{st}^* = \sum_{h=1}^H \{w_h + w_h q_h (\lambda_1 x_{hj1} + \lambda_2 x_{hj2} + \cdots + \lambda_m x_{hjm})\} y_{hj} \quad (17)$$

The determination of the optimum weights w_h^* using the Lagrange's multipliers technique discussed in this section is illustrated in the following theorem when information on three auxiliary variables X_i ($i = 1, 2, 3$) is available.

Theorem

In stratified sampling, given three auxiliary variables, the optimum calibration weights w_h^* that minimize

$$Z = \sum_{h=1}^H \frac{w_h}{q_h} \left(\frac{w_h^*}{w_h} - 1 \right)^2$$

Subject to

$$\begin{aligned} \sum_{h=1}^H w_h^* x_{hj1} &= X_1 \\ \sum_{h=1}^H w_h^* x_{hj2} &= X_2 \\ \sum_{h=1}^H w_h^* x_{hj3} &= X_3 \end{aligned}$$

is given by

$$w_h^* = w_h + w_h q_h (\lambda_1 x_{hj1} + \lambda_2 x_{hj2} + \lambda_3 x_{hj3})$$

where

$$\begin{aligned} q_h &= \sum_{i=1}^m q_{hi}, w_h^* \geq 0; h = 1, 2, \dots, H; j = 1, 2, \dots, N_h \\ \lambda_1 &= \frac{P(T^2 - UR)(X_1 - \hat{X}_1) + P(QU - ST)(X_2 - \hat{X}_2) + P(RS - QT)(X_3 - \hat{X}_3)}{(QS - PT)^2 - (S^2 - PU)(Q^2 - PR)} \\ \lambda_2 &= \frac{P(QU - ST)(X_1 - \hat{X}_1) + P(S^2 - PU)(X_2 - \hat{X}_2) + P(QS - PT)(X_3 - \hat{X}_3)}{(QS - PT)^2 - (S^2 - PU)(Q^2 - PR)} \\ \lambda_3 &= \frac{P(RS - QT)(X_1 - \hat{X}_1) - P(QS - PT)(X_2 - \hat{X}_2) - P(Q^2 - PR)(X_3 - \hat{X}_3)}{(QS - PT)^2 - (S^2 - PU)(Q^2 - PR)} \\ P &= \sum_{h=1}^H w_h q_h x_{hj1}^2; Q = \sum_{h=1}^H w_h q_h x_{hj1} x_{hj2}; R = \sum_{h=1}^H w_h q_h x_{hj2}^2; \\ S &= \sum_{h=1}^H w_h q_h x_{hj1} x_{hj3}; T = \sum_{h=1}^H w_h q_h x_{hj2} x_{hj3}; U = \sum_{h=1}^H w_h q_h x_{hj3}^2 \end{aligned}$$

$$\hat{X}_1 = \sum_{h=1}^H w_h x_{hj1}; \quad \hat{X}_2 = \sum_{h=1}^H w_h x_{hj2}; \quad \hat{X}_3 = \sum_{h=1}^H w_h x_{hj3}$$

Proof of Theorem

Using the Lagrange's multiplier technique, the function to be minimized is:

$$\begin{aligned} L(w_h^*, \lambda_i) = & \sum_{h=1}^H \frac{w_h}{q_h} \left(\frac{w_h^*}{w_h} - 1 \right)^2 - 2\lambda_1 \left(\sum_{h=1}^H w_h^* x_{hj1} - X_1 \right) - 2\lambda_2 \left(\sum_{h=1}^H w_h^* x_{hj2} - X_2 \right) \\ & - 2\lambda_3 \left(\sum_{h=1}^H w_h^* x_{hj3} - X_3 \right) \end{aligned} \quad (18)$$

The necessary conditions as given in equation (12) are:

$$\frac{\partial L}{\partial w_h^*} = 2(w_h^* - w_h) - 2\lambda_1 w_h q_h x_{hj1} - 2\lambda_2 w_h q_h x_{hj2} - 2\lambda_3 w_h q_h x_{hj3} = 0$$

$$w_h^* = w_h + \lambda_1 w_h q_h x_{hj1} + \lambda_2 w_h q_h x_{hj2} + \lambda_3 w_h q_h x_{hj3} \quad (19)$$

$$\frac{\partial L}{\partial \lambda_1} = -2 \left(\sum_{h=1}^H w_h^* x_{hj1} - X_1 \right) = 0 \Rightarrow \sum_{h=1}^H w_h^* x_{hj1} = X_1 \quad (20)$$

$$\frac{\partial L}{\partial \lambda_2} = -2 \left(\sum_{h=1}^H w_h^* x_{hj2} - X_2 \right) = 0 \Rightarrow \sum_{h=1}^H w_h^* x_{hj2} = X_2 \quad (21)$$

$$\begin{aligned} & \vdots \\ \frac{\partial L}{\partial \lambda_3} = & -2 \left(\sum_{h=1}^H w_h^* x_{hj3} - X_3 \right) = 0 \Rightarrow \sum_{h=1}^H w_h^* x_{hj3} = X_3 \end{aligned} \quad (22)$$

Solving the necessary conditions (19) to (22) by substituting (19) into (20), (21) and (22) respectively gives:

$$\lambda_1 \sum_{h=1}^H w_h q_h x_{hj1}^2 + \lambda_2 \sum_{h=1}^H w_h q_h x_{hj1} x_{hj2} + \lambda_3 \sum_{h=1}^H w_h q_h x_{hj1} x_{hj3} = X_1 - \hat{X}_1$$

$$\lambda_1 \sum_{h=1}^H w_h q_h x_{hj1} x_{hj2} + \lambda_2 \sum_{h=1}^H w_h q_h x_{hj2}^2 + \lambda_3 \sum_{h=1}^H w_h q_h x_{hj2} x_{hj3} = X_2 - \hat{X}_2$$

$$\lambda_1 \sum_{h=1}^H w_h q_h x_{hj1} x_{hj3} + \lambda_2 \sum_{h=1}^H w_h q_h x_{hj2} x_{hj3} + \lambda_3 \sum_{h=1}^H w_h q_h x_{hj3}^2 = X_3 - \hat{X}_3$$

and solving the above simultaneous equations completes the proof.

3. Simulation and Discussion

We considered an artificial population taken from Cochran (1977; Table 6.1). The x -variable represents the auxiliary variables while the y -variable represents the study variable. The population was

divided into three strata with three cells each with six units selected by systematic sampling scheme. Three units were then selected from the six units earlier selected by simple random sampling without replacement. Therefore, 27 units were selected from 49 units in the original population as presented in table 1.

Table 1: Population adapted from Cochran (1977)

Stratum1				Stratum2				Stratum3			
x_{hj1}	x_{hj2}	x_{hj3}	y_{hj}	x_{hj1}	x_{hj2}	x_{hj3}	y_{hj}	x_{hj1}	x_{hj2}	x_{hj3}	y_{hj}
76	46	46	514	23	50	71	612	78	56	43	560
67	2	87	645	37	44	43	423	66	44	36	548
29	121	30	623	61	64	25	760	60	38	74	585
172	169	163	1,782	121	158	139	1,805	204	138	153	1,693

Table 2 shows the following sample information $\hat{X}_1 = \sum_{h=1}^H w_h x_{hj1} = 171.5321$;

$\hat{X}_2 = \sum_{h=1}^H w_h x_{hj2} = 153.4275$; $\hat{X}_3 = \sum_{h=1}^H w_h x_{hj3} = 152.5508$;

$P = \sum_{h=1}^H w_h q_h x_{hj1}^2 = 30,531.0845$; $Q = \sum_{h=1}^H w_h q_h x_{hj1} x_{hj2} = 26,054.3538$;

$R = \sum_{h=1}^H w_h q_h x_{hj2}^2 = 23,721.8765$; $S = \sum_{h=1}^H w_h q_h x_{hj1} x_{hj3} = 26,356.5731$;

$T = \sum_{h=1}^H w_h q_h x_{hj2} x_{hj3} = 23,439.3489$; $U = \sum_{h=1}^H w_h q_h x_{hj3}^2 = 23,356.1936$

Table 2: Sample information

estimator	Stratum 1	Stratum 2	Stratum 3	total
N_h	16	13	20	$N = 49$
n_h	9	9	9	$n = 27$
x_{hj1}	172	121	204	-
x_{hj2}	169	158	138	-
x_{hj3}	163	139	153	-
y_{hj}	1782	1805	1693	-
w_h	0.3265	0.2653	0.4082	
$w_h x_{hj1}$	52.1580	32.1013	83.2728	171.5321
$w_h x_{hj2}$	55.1785	41.9174	56.3316	153.4275
$w_h x_{hj3}$	53.2195	36.8767	62.4546	152.5508
$w_h q_h x_{hj1}^2$	9659.1760	3884.2573	16987.6512	30531.0845
$w_h q_h x_{hj1} x_{hj2}$	9490.7020	5072.0054	1149.6464	26054.3538
$w_h q_h x_{hj2}^2$	9325.1665	6622.9492	7773.7608	23721.8765
$w_h q_h x_{hj1} x_{hj3}$	9153.7540	4462.0807	12740.7384	26356.5731
$w_h q_h x_{hj2} x_{hj3}$	8994.0955	5826.5186	8618.7348	23439.3489
$w_h q_h x_{hj3}^2$	8674.7785	5125.8613	9555.5538	23356.1936
$w_h y_{hj}$	581.8230	478.8665	691.0826	1751.7721

For the population the known population totals for the auxiliary variables are $X_1 = 497$; $X_2 = 465$; $X_3 = 455$ and from the analysis $\lambda_1 = 0.2308$; $\lambda_2 = 0.4996$, $\lambda_3 = -0.7452$.

Thus, the multivariate calibrated weights (w_h^*) in stratified sampling proposed in (19) reduces to

$$w_h^* = w_h + w_h(0.2308x_{hj1} + 0.4996x_{hj2} - 0.7452x_{hj3}) \quad (23)$$

which are obtained and presented in table 3.

The conventional estimator of domain total in stratified sampling given in (4) is

$$y_{st} = \sum_{h=1}^H w_h y_{hj} = 1,751.7721 \quad (24)$$

Whereas an estimate using the proposed generalized multivariate estimator in (5) is

$$y_{st}^* = \sum_{h=1}^H w_h^* y_{hj} = 6,262.3798 \quad (25)$$

The true total for this population is 6,262 (see Cochran 1977 pp.152). Therefore from (24) and (25), it is evident that the proposed multivariate calibration estimator is a better approximation of the true population total than the conventional estimator.

Table 3: Optimum calibrated weights

stratum	1	2	3
Optimum weights (w_h^*)	1.1956	1.1357	1.2297

4. Conclusions

Analytical approach for determining multivariate calibration estimator to improve survey estimates when multiple auxiliary variables are available is developed. The problem of determining the optimum calibration weights was formulated as a Mathematical Programming Problem (MPP) that minimizes the Chi-square type loss function subject to multiple calibration constraints using Lagrange multiplier technique. The optimum calibration weights do not violate the calibration restrictions. An empirical study has been given to show the performance of the proposed multivariate calibration estimator over the stratified random sampling estimator. The results showed that the multivariate calibration estimator is more efficient than the conventional stratified random sampling estimator.

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