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# A New Method for Finding an Optimal Solution of Assignment Problem

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**Abstract:** In this paper a new method is proposed for finding an optimal solution of a wide range of assignment problems, directly. A numerical illustration is established and the optimality of the result yielded by this method is also checked. The most attractive feature of this method is that it requires very simple arithmetical and logical calculations. The method is illustrated through an example.

**Keywords**: Assignment problem, Transportation problem, Hungarian Method.

**Mathematical Subject Classifications (2000):** 68M15, 62F15

## 1. Introduction

The Assignment Problem is one of the fundamental combinatorial optimization problems in the branch of optimization (or operations research) in mathematics. It is a particular case of transportation problem where the sources are assignees and the destinations are tasks. In this problem every source has a supply one (since each assignee is to be assigned exactly one task) and every destination has a demand one (since each task is to be performed by exactly one assignee). The assignments are to be made in such a way so as to maximize (or minimize) the total effectiveness. Thus in an assignment problem the number of assignees must be equal to the number of tasks so that each assignee has one and only one task. The problem of assignment arises because the resources that are available such as men, machines etc. have varying degrees of efficiency for performing different activities. Therefore cost, profit or time of performing different activities is also different. Thus the problem is: how should the assignments be made so as to optimize the given objective?

The problem owes its name to the classical application of assigning a number of jobs to an equal number of persons at a minimal total cost, given the costs of the assignment of every job to every person. It is evident that even if every person is capable of employing every job, the solution of the problem must assign only one job to each person. From a first point of view it looks really simple; one must just find all the possible combinations between the jobs and the persons, compare the respective costs, and choose the minimal one. However, taking into account that there are n! possible permutations of n discrete objects, the approach just mentioned is computationally prohibitive, considering that the actual problem can be large. The assignment problem finds many applications in allocation and scheduling e.g. In assigning salesman to different regions, jobs to machines, clerks to various checkout counters, products to factories etc.

The first specialized method for the Assignment Problem was given by H.W.Kuhn [1] in 1955 and was subsequently extended for the solution of much more general network flow problems. In 1981 D.P. Bertsekas [2] studied the problem who gave new Algorithm for the solution of the problem. In 2007 S.Anshuman and T.Rudrajit [4] solve Assignment Problem using Genetic Algorithm and Simulated Annealing. Here a much easier heuristic approach is proposed for finding an optimal solution with lesser number of iterations and very easy computations.

## 2. Non Standard Assignment Problem

In case if the number of assignees should not be equal to the number of tasks i.e. ($M \neq N$) we have to convert this problem into standard assignment problem with equal number of assignees and tasks. For this we create dummy assignees or dummy tasks. If assignees is more than tasks ($M > N$), we then create ($M – N$) dummy tasks so that there will be M assignees and N tasks. The assigning cost of the dummy task to be zero so that the objective function will be unaltered. Similarly when we have more tasks than assignees ($N > M$), we create ($N – M$) dummy assignees whose cost will be zero.

## 3. Formulation of the Problem

"Given N men and N machines, we have to assign each single machine to a single man in such a manner that the overall cost of assignment is minimized." To put it mathematically, define the following symbols:

$i \rightarrow$row number denoting $i^{th}$ machine $i \in [1, N]$

$j \rightarrow$column number denoting $j^{th}$ men $j \in [1, N]$

$C \rightarrow$ cost of assigning $i^{th}$ machine to $j^{th}$ man

$X_{ij} = 1$ if $i^{th}$ machine is assigned to $j^{th}$ man

$= 0$ otherwise

The problem can be formulated as:

Minimize the total cost function $\sum_{i=1}^{N}\sum_{j=1}^{N} c_{ij} x_{ij}$

Subject to the following constraints

$$\sum_{j=1}^{N} x_{ij} = 1, \qquad i = 1,2,\ldots,N \tag{1}$$

$$\sum_{i=1}^{N} x_{ij} = 1, \qquad j = 1,2,\ldots,N \tag{2}$$

$$x_{ij} = 1 \, or \, 0 \tag{3}$$

## 4. Algorithm of New Method

Now, we introduce a new algorithm for finding optimal solution of an Assignment problem. The step wise procedure of the proposed method is as follows:

Step 1: Construct the Assignment table from the given Assignment problem.

Step 2: Prepare a square matrix. This step will not require for N x N Assignment problem. For M x N problem a dummy column or dummy row, as the case may be, is added to make the square matrix.

Step 3: Subtract the smallest element of each row from every element of the respective row and then subtract smallest element of each column of the reduced matrix from all the elements of the respective column.

Step 4: Now there will be at least one zero in each row and in each column in the reduced matrix. Select the first zero (row-wise) occurring in the cost matrix. Suppose $(i, j)^{th}$ zero is selected , count the total number of zero's in the $i^{th}$ row and $j^{th}$ column. Now select the next zero and count the total number of zero's in the corresponding row and column in the same manner. Continue it for all zero's in the matrix.

Step 5: Now select a zero for which the number of zero's counted in above step is minimum. Make an Assignment to this cell by making square ( $\square$ ) around it. If tie occurs it can be broken by choosing (i, j)$^{th}$ zero breaking tie such that total sum of all the elements in the i$^{th}$ row and j$^{th}$ column is maximum. Make an assignment to this cell. If again tie occurs it can be broken by selecting the minimum cost cell.

Step 6: After performing step 4, delete the row and column for further calculation as they will not be considered for making any more assignments.

Step 7: Check whether the resultant matrix possesses at least one zero in each row and in each column. If not, repeat 3. Otherwise go to step 8.

Step 8: Repeat steps 4 to 7 until and unless each row and each column have one assignment.

## 5. Numerical Example

Consider the following cost minimizing Assignment problem with five jobs and five machines.

Table 1

| | $M_1$ | $M_2$ | $M_3$ | $M_4$ | $M_5$ |
|---|---|---|---|---|---|
| $J_1$ | 5 | 5 | 7 | 4 | 8 |
| $J_2$ | 6 | 5 | 8 | 3 | 7 |
| $J_3$ | 6 | 8 | 9 | 5 | 10 |
| $J_4$ | 7 | 6 | 6 | 3 | 6 |
| $J_5$ | 6 | 7 | 10 | 6 | 11 |

After applying this new method the assignments are obtained as follows

Table 2

| | $M_1$ | $M_2$ | $M_3$ | $M_4$ | $M_5$ |
|---|---|---|---|---|---|
| $J_1$ | 5 | 5 | $\boxed{7}$ | 4 | 8 |
| $J_2$ | 6 | $\boxed{5}$ | 8 | 3 | 7 |

| | | | | | |
|---|---|---|---|---|---|
| $J_3$ | 6 | 8 | 9 | $\boxed{5}$ | 10 |
| $J_4$ | 7 | 6 | 6 | 3 | $\boxed{6}$ |
| $J_5$ | $\boxed{6}$ | 7 | 10 | 6 | 11 |

As there is one and only one assignment in each row and column, thus optimality can be made in solution. The optimal assignment policy is

Job $J_1$ should be assigned to machine $M_3$

Job $J_2$ should be assigned to machine $M_2$

Job $J_3$ should be assigned to machine $M_4$

Job $J_4$ should be assigned to machine $M_5$

Job $J_5$ should be assigned to machine $M_1$

The total cost associated with these assignments Rs 29.

## 6. Optimality Check

To find whether the solution obtained is optimal or not we apply Hungarian Method for the above problem. And after applying the Hungarian method the total cost of the problem is Rs 29. It can be seen that value of objective function obtained by our method is same as that of Hungarian Method. Hence the solution obtained by our method is also optimal.

## 7. Conclusion

Thus it can be concluded that our method provides an optimal solution in fewer iterations, for the solution of an Assignment Problem. As this method consumes less time and is very easy to understand and apply, so it will be very helpful for decision makers who are dealing with logistic and supply chain problems. The future research work may be considered to introduce the mathematical formulation of the proposed method and algorithm.

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