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Article

Modified Ratio Estimator for Estimating Population Mean Using Auxiliary Information in Survey Sampling

[1]Mir Subzar*, [1]S. Maqbool, [1]N A Sofi, [1]T A Raja and [2]Zaffar Bashir

[1]*Division of Agricultural Statistics, SKUAST-Kashmir (190025), India*

[2]*Division of Basic Sciences and Humanities (Wadura), SKUAST-Kashmir (190025), India*

*Author to whom correspondence should be addressed; E-Mail: subzarstat@gmail.com

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Abstract: In this paper we propose a family of modified ratio type estimator for estimating the population mean of the study variable using the auxiliary information of population standard deviation, coefficient of skewness, correlation coefficient, coefficient of kurtosis and coefficient of variation. The properties associated with the proposed estimator is evaluated through MSE and bias. We also provide an empirical study for illustration and verification.

Keywords: Auxiliary information; Ratio-type estimators; Mean square error; Bias; Efficiency

Mathematics Subject Classification: 62D05

1. Introduction

In sample surveys, auxiliary information is used at selection as well as estimation stages to improve the design as well as obtaining more efficient estimators. Increased precision can be obtained when study variable Y is correlated with auxiliary variable X and the ratio estimator is used the correlation between the study and the auxiliary variable is positively correlated. Cochran [1] suggested a classical ratio type estimator for the estimation of finite population mean using one auxiliary variable under simple random sampling scheme. Upadhyaya & Singh [8] modified ratio type estimators using coefficient of variation and coefficient of kurtosis of the auxiliary variable. Singh & Tailor [5] proposed a family of estimators using known values of some parameters in srswor for estimation of population mean of the study variable. Sisodia & Dwivedi [7] and Singh *et al.* [6] utilized coefficient of variation of the auxiliary variable to obtain the estimates with more precision and the significance of the present paper deals with the proposing such estimators which are more efficient than the existing estimators for estimating the population mean of the study variable. Consider a finite population $U=\{U_1,U_2,U_3,\ldots,U_N\}$ of N distinct and identifiable units. Let Y be the study variable with value Y_i measured of U_i, $i=1,2,3,\ldots,N$ giving a vector $Y=\{Y_1,Y_2,Y_3,\ldots,Y_N\}$. The objective is to estimate population mean $\bar{Y}=\frac{1}{N}\sum_{i=1}^{N}Y_i$ on the basis of a random sample. The mean ratio estimator for estimating the population mean, $\bar{Y}$, of the study variable Y is defined as

$$\bar{Y}_r=\frac{\bar{y}}{\bar{x}}\bar{X}$$

The bias, related constant and the mean squared error (MSE) of the ratio estimator are respectively given by

$$B(\bar{Y}_r)=\frac{(1-f)}{n}\frac{1}{\bar{X}}(RS_x^2-\rho S_xS_Y)\quad R=\frac{\bar{Y}}{\bar{X}}\quad MSE(\bar{Y}_r)=\frac{1-f}{n}(S_y^2+R^2S_x^2-2R\rho S_xS_Y)$$

2. Existing Ratio Estimators

Kadilar and Cingi [2] suggested ratio type estimators for the population mean in the simple random sampling using some known auxiliary information on coefficient of kurtosis and coefficient of variation. They showed that their suggested estimators are more efficient than traditional ratio estimator in the estimation of the population mean.

Kadilar & Cingi [2] estimators are given by

$$\bar{Y}_1=\frac{\bar{y}+b(\bar{X}-\bar{x})}{\bar{x}}\bar{X},\ \bar{Y}_2=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+C_x)}(\bar{X}+C_x),\ \bar{Y}_3=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+\beta_2)}(\bar{X}+\beta_2),$$

$$\bar{Y}_4=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\beta_2+C_x)}(\bar{X}\beta_2+C_x),\ \bar{Y}_5=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+\beta_2)}(\bar{X}C_x+\beta_2),$$

The biases, related constants and the MSE for Kadilarand Cingi [2] estimators are respectively as follows:

$$B(\bar{Y}_1)=\frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_1^2,\quad R_1=\frac{\bar{Y}}{\bar{X}}\qquad MSE(\bar{Y}_1)=\frac{(1-f)}{n}(R_1^2S_x^2+S_y^2(1-\rho^2)),$$

$$B(\bar{Y}_2) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_2^2, \quad R_2 = \frac{\bar{Y}}{(\bar{X}+C_x)} \qquad MSE(\bar{Y}_2) = \frac{(1-f)}{n}(R_2^2 S_x^2 + S_y^2(1-\rho^2)),$$

$$B(\bar{Y}_3) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_3^2, \quad R_3 = \frac{\bar{Y}}{(\bar{X}+\beta_2)} \qquad MSE(\bar{Y}_3) = \frac{(1-f)}{n}(R_3^2 S_x^2 + S_y^2(1-\rho^2)),$$

$$B(\bar{Y}_4) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_4^2, \quad R_4 = \frac{\bar{Y}}{(\bar{X}\beta_2+C_x)} \qquad MSE(\bar{Y}_4) = \frac{(1-f)}{n}(R_4^2 S_x^2 + S_y^2(1-\rho^2)),$$

$$B(\bar{Y}_5) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_5^2, \quad R_5 = \frac{\bar{Y}}{(\bar{X}C_x+C_x)} \qquad MSE(\bar{Y}_5) = \frac{(1-f)}{n}(R_5^2 S_x^2 + S_y^2(1-\rho^2)).$$

Kadilar and Cingi [3] developed some modified ratio estimators using known value of coefficient of correlation, kurtosis and coefficient of variation as follows:

$$\bar{Y}_6 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+\rho)}(\bar{X}+\rho), \bar{Y}_7 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+\rho)}(\bar{X}C_x+\rho), \bar{Y}_8 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\rho+C_x)}(\bar{X}\rho+C_x),$$

$$\bar{Y}_9 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\beta_2+\rho)}(\bar{X}\beta_2+\rho),$$

$$\bar{Y}_{10} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\rho+\beta_2)}(\bar{X}\rho+\beta_2).$$

The biases, related constants and the MSE for Kadilar and Cingi [3] estimators are respectively given by

$$B(\bar{Y}_6) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_6^2, \quad R_6 = \frac{\bar{Y}}{\bar{X}+\rho} \qquad MSE(\bar{Y}_6) = \frac{(1-f)}{n}(R_6^2 S_x^2 + S_y^2(1-\rho^2)),$$

$$B(\bar{Y}_7) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_7^2, \quad R_7 = \frac{\bar{Y}C_x}{\bar{X}C_x+\rho} \qquad MSE(\bar{Y}_7) = \frac{(1-f)}{n}(R_7^2 S_x^2 + S_y^2(1-\rho^2)),$$

$$B(\bar{Y}_8) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_8^2, \quad R_8 = \frac{\bar{Y}\rho}{\bar{X}\rho+C_x} \qquad MSE(\bar{Y}_8) = \frac{(1-f)}{n}(R_8^2 S_x^2 + S_y^2(1-\rho^2)),$$

$$B(\bar{Y}_9) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_9^2, \quad R_9 = \frac{\bar{Y}\beta_2}{\bar{X}\beta_2+\rho} \qquad MSE(\bar{Y}_9) = \frac{(1-f)}{n}(R_9^2 S_x^2 + S_y^2(1-\rho^2)),$$

$$B(\bar{Y}_{10}) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_{10}^2, \quad R_{10} = \frac{\bar{Y}\rho}{\bar{X}\rho+\beta_2} \qquad MSE(\bar{Y}_{10}) = \frac{(1-f)}{n}(R_{10}^2 S_x^2 + S_y^2(1-\rho^2)).$$

3. Proposed Modified Ratio Estimator

Motivated by the mentioned estimators in Section 2, we propose new family of efficient ratio type estimators using auxiliary information in survey sampling. The proposed estimators is given below

$$\bar{Y}_{p1} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\rho+S_x)}(\bar{X}\rho+S_x).$$

$$\bar{Y}_{p2} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\beta_2 + S_x)}(\bar{X}\beta_2 + S_x).$$

$$\bar{Y}_{p3} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\beta_1 + S_x)}(\bar{X}\beta_1 + S_x).$$

$$\bar{Y}_{p4} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}C_x + S_x)}(\bar{X}C_x + S_x).$$

The bias, related constant and the MSE for the first proposed estimator can be obtained as follows: MSE of this estimator can be found using Taylor series method defined as

$$h(\bar{x}, \bar{y}) \cong h(\bar{X}, \bar{Y}) + \frac{\partial h(c,d)}{\partial c}|_{\bar{X},\bar{Y}} (\bar{x} - \bar{X}) + \frac{\partial h(c,d)}{\partial d}|_{\bar{X},\bar{Y}} (\bar{y} - \bar{Y}) \tag{3.1}$$

Where $h(\bar{x}, \bar{y}) = \hat{R}_{p1}$ and $h(\bar{X}, \bar{Y}) = R.$

As shown in Wolter [9], (3.1) can be applied to the proposed estimators in order to obtain MSE equation as follows:

$$\hat{R}_{p1} - R \cong \frac{\partial((\bar{y} + b(\bar{X} - \bar{x}))/(\bar{x}\rho + S_x)}{\partial \bar{x}}|_{\bar{X},\bar{Y}} (\bar{x} - \bar{X}) + \frac{\partial((\bar{y} + b(\bar{X} - \bar{x}))/(\bar{x}\rho + S_x)}{\partial \bar{y}}|_{\bar{X},\bar{Y}} (\bar{y} - \bar{Y})$$

$$\cong -\left(\frac{\bar{y}}{(\bar{x}\rho + S_x)^2} + \frac{b(\bar{X}\rho + S_x)}{(\bar{x}\rho + S_x)^2}\right)|_{\bar{X},\bar{Y}} (\bar{x} - \bar{X}) + \frac{1}{(\bar{x}\rho + S_x)}|_{\bar{X},\bar{Y}} (\bar{y} - \bar{Y})$$

$$E(\hat{R}_{p1} - R)^2 \cong \frac{(\bar{Y}\rho + B(\bar{X}\rho + S_x))^2}{(\bar{X}\rho + S_x)^4}V(\bar{x}) - \frac{2(\bar{Y}\rho + B(\bar{X}\rho + S_x))}{(\bar{X}\rho + S_x)^3}Cov(\bar{x}, \bar{y}) + \frac{1}{(\bar{X}\rho + S_x)^2}V(\bar{y})$$

$$\cong \frac{1}{(\bar{X}\rho + S_x)^2}\left\{\frac{(\bar{Y}\rho + B(\bar{X}\rho + S_x))^2}{(\bar{X}\rho + S_x)^2}V(\bar{x}) - \frac{2(\bar{Y}\rho + B(\rho + S_x)}{(\bar{X}\rho + S_x)}Cov(\bar{x}, \bar{y}) + V(\bar{y})\right\}$$

Where $B = \frac{s_{xy}}{s_x^2} = \frac{\rho s_x s_y}{s_x^2} = \frac{\rho s_y}{s_x}$. Note that we omit the difference of $(E(b) - B)$.

$$MSE(\bar{y}_{p1}) = (\bar{X}\rho + S_x)^2 E(\hat{R}_{pj} - R)^2 \cong \frac{(\bar{Y}\rho + B(\bar{X}\rho + S_x))^2}{(\bar{X}\rho + S_x)^2}V(\bar{x}) - \frac{2(\bar{Y}\rho + B(\bar{X}\rho + S_x))}{(\bar{X}\rho + S_x)}Cov(\bar{x}, \bar{y}) + V(\bar{y})$$

$$\cong \frac{\bar{Y}^2\rho + 2B(\bar{X}\rho + S_x)\bar{Y}\rho + B^2(\bar{X}\rho + S_x)^2}{(\bar{X}\rho + S_x)^2}V(\bar{x}) - \frac{2\bar{Y}\rho + 2B(\bar{X}\rho + S_x)}{(\bar{X}\rho + S_x)}Cov(\bar{x}, \bar{y}) + V(\bar{y})$$

$$\cong \frac{(1-f)}{n}\left\{\left(\frac{\bar{Y}^2\rho}{(\bar{X}\rho + S_x)^2} + \frac{2B\bar{Y}\rho}{(\bar{X}\rho + S_x)} + B^2\right)S_x^2 - \left(\frac{2\bar{Y}\rho}{(\bar{X}\rho + S_x)} + 2B\right)S_{xy} + S_y^2\right\}$$

$$\cong \frac{(1-f)}{n}(R^2S_x^2 + 2BRS_x^2 + B^2S_x^2 - 2RS_{xy} - 2BS_{xy} + S_y^2)$$

$$MSE(\bar{y}_{pj}) \cong \frac{(1-f)}{n}(R^2S_x^2 + 2R\rho S_x S_y + \rho^2 S_y^2 - 2R\rho S_x S_y - 2\rho^2 S_y^2 + S_y^2$$

$$\cong \frac{(1-f)}{n}(R^2S_x^2 - \rho^2 S_y^2 + S_y^2) \cong \frac{(1-f)}{n}(R^2S_x^2 + S_y^2(1 - \rho^2)$$

Similarly, the bias is obtained as

$$Bias(\bar{y}_{pj}) \cong \frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_j^2$$

Thus the bias and MSE of the proposed estimators is given below:

$$B(\bar{Y}_{p1}) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_1^2, \quad R_1 = \frac{\bar{Y}\rho}{\bar{X}\rho + S_x} \qquad MSE(\bar{Y}_{p1}) = \frac{(1-f)}{n}(R_1^2 S_x^2 + S_y^2(1-\rho^2)),$$

Where $j = 1,2,\ldots,10$ and $k = 1,2,\ldots,12$.

Similarly the Constant, bias and MSE for other proposed estimator can be obtained as follows:

$$B(\bar{Y}_{p2}) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_2^2, \qquad R_2 = \frac{\bar{Y}\beta_2}{\bar{X}\beta_2 + S_x} \qquad MSE(\bar{Y}_{p2}) = \frac{(1-f)}{n}(R_2^2 S_x^2 + S_y^2(1-\rho^2)).$$

$$B(\bar{Y}_{p3}) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_3^2, \qquad R_3 = \frac{\bar{Y}\beta_1}{\bar{X}\beta_1 + S_x} \qquad MSE(\bar{Y}_{p3}) = \frac{(1-f)}{n}(R_3^2 S_x^2 + S_y^2(1-\rho^2)).$$

$$B(\bar{Y}_{p4}) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_4^2, \qquad R_4 = \frac{\bar{Y}C_x}{\bar{X}C_x + S_x} \qquad MSE(\bar{Y}_{p4}) = \frac{(1-f)}{n}(R_4^2 S_x^2 + S_y^2(1-\rho^2)).$$

4. Efficiency Comparisons

Comparisons with existing ratio estimators

From the expressions of the MSE of the proposed estimators and the existing estimators, we have derived the conditions for which the proposed estimators are more efficient than the existing modified ratio estimators as follows:

$$MSE(\bar{Y}_{pj}) \le MSE(\bar{Y}_i),$$

$$\frac{(1-f)}{n}(R_{pj}^2 S_x^2 + S_y^2(1-\rho^2)) \le \frac{(1-f)}{n}(R_i^{\,2} S_x^2 + S_y^2(1-\rho^2)),$$

$$R_{pj}^2 S_x^2 \le R_i^2 S_x^2,$$

$$R_{pj} \le R_i,$$

Where $j = 1,2,\ldots.,4$ and $i = 1,2,\ldots.,10$.

5. Empirical Study

The performances of the suggested ratio estimators are evaluated and compared with the mentioned ratio estimators in Section 2 by using natural Population.

The percentage relative efficiency (PREs) of the proposed estimators (p), with respective to the existing estimators (e), are computed as

$$PRE = \frac{MSE\,of\ Existing\ Estimator}{MSE\,of\ propoesd\ estimator} \times 100$$

The Population is taken from page 177 of Singh and Chaudhary [4].

The statistics of population taken from Singh and Chaudhary [4] is given in Table 1:

Table 1: Data Statistics

$N = 34$	$S_y = 733.1407$
$n = 20$	$C_y = 0.8561$
$\bar{Y} = 856.4117$	$C_x = 0.7205$
$\bar{X} = 208.8823$	$\beta_1 = 0.9782$
$S_x = 150.5059$	$\beta_2 = 0.0978$
$\rho = 0.4491$	$Md = 150$

Table 2: Constants, Bias and the MSE of the existing and the proposed estimators

Estimators	**Constant**	**Bias**	**MSE**
$\bar{Y}_1$	4.100	9.154	16673.45
$\bar{Y}_2$	4.086	9.091	16619.64
$\bar{Y}_3$	4.098	9.145	16666.14
$\bar{Y}_4$	3.960	8.539	16146.61
$\bar{Y}_5$	4.097	9.142	16663.31
$\bar{Y}_6$	4.091	9.115	16639.85
$\bar{Y}_7$	4.088	9.099	16626.87
$\bar{Y}_8$	4.069	9.015	16554.4
$\bar{Y}_9$	4.012	8.763	16338.65
$\bar{Y}_{10}$	4.096	9.135	16657.19
$\bar{Y}_{p1}$	**1.574**	**1.349**	**9989.93**
$\bar{Y}_{p2}$	**0.489**	**0.130**	**8946.12**
$\bar{Y}_{p3}$	**2.360**	**3.033**	**11433.68**
$\bar{Y}_{p4}$	**2.049**	**2.288**	**10793.94**

Table 3: PRE of proposed estimators with respect to existing estimators

	$\bar{Y}_{p1}$	$\bar{Y}_{p2}$	$\bar{Y}_{p3}$	$\bar{Y}_{p4}$
$\bar{Y}_1$	166.90	186.37	145.57	154.47
$\bar{Y}_2$	166.36	185.77	145.10	153.97
$\bar{Y}_3$	166.82	186.29	145.50	154.40
$\bar{Y}_4$	161.62	180.48	140.97	149.59
$\bar{Y}_5$	166.80	186.26	145.48	154.37
$\bar{Y}_6$	166.56	186.00	145.27	154.15
$\bar{Y}_7$	166.43	185.85	145.16	154.03
$\bar{Y}_8$	165.71	185.04	144.53	153.36
$\bar{Y}_9$	163.55	182.63	142.64	151.36
$\bar{Y}_{10}$	166.73	186.19	145.43	154.32

6. Conclusion

From the above empirical study it is evident that the proposed family of the ratio type estimator using the auxiliary information of population standard deviation and coefficient of skewness , coefficient of kurtosis , coefficient of skewness and correlation coefficient are more efficient than the existing estimators as MSE & Bias of the proposed estimator is lower than the existing estimator. From the discussion point of view about these proposed estimators and the one which is performing better than among all these estimators is the fourth one but in fact all these are performing well than the already existing estimators in the literature. Hence we strongly recommend that the proposed estimators preferred over the existing estimators to use in practical application for future studies.

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