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Article

The Inverse Weibull-Geometric Distribution

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Abstract: Inverse Weibull-Geometric Distribution which generalizes the Inverse Exponential-Geometric distribution, Inverse Weibull distribution, Inverse Exponential distribution and Inverse Rayleigh distribution has been introduced in this paper. The model can be considered as another useful 3-parameter distribution. Model characterization is studied, we derive the cumulative distribution and hazard functions, the density of the order statistics and calculate expressions for its moments and for the moments of the order statistics. Mixture model of two Inverse Weibull-Geometric distributions is investigated. Estimates of parameters using method of maximum likelihood have been obtained through simulations. Two real life example are provided one for complete data another for censored data to show the flexibility and potentiality of the proposed distribution and comparison with Inverse Weibull distribution, Inverse Exponential Geometric distribution, Inverse Exponential distribution and Inverse Rayleigh distribution is also discussed. The proposed model compares well with other competing models to fit the data.

Keywords: Inverse Weibull distribution, Inverse Rayleigh distribution Hazard rate, Mixture distribution, Censored data.

Mathematics Subject Classification Code (2010): 68M15, 62F15

1. Introduction

The inverse Weibull distribution is one of the most commonly used lifetime distribution which can be used in the reliability engineering discipline. The Inverse Weibull distribution is used to model a variety of failure characteristics such as infant mortality, useful life and wear-out periods and can be used to determine the cost effectiveness and maintenance periods of reliability centered maintenance activities. In literature various other distributions have been proposed to model lifetime model. Adamidis and Loukas [2] proposed the two-parameter exponential-geometric distribution with decreasing failure rate. Kus [13] introduced the exponential-Poisson distribution with decreasing failure rate and discussed several of its properties. Adamidis, Dimitrakopoulou and Loukas [1] proposed the extended exponential-geometric distribution which generalizes the exponential-geometric distribution. The hazard function of the extended exponential-geometric distribution can be monotone decreasing, increasing or constant and they discussed several reliability features and its properties. Wagner, Alice and Gauss [4] proposed the Weibull-Geometric distribution which generalizes the exponential-geometric distribution proposed by Adamidis, Dimitrakopoulou and Loukas. The hazard function of Weibull-Geometric distribution takes more general forms. The Weibull-Geometric distribution is useful for modeling unimodal failure rates. Wang and Elbatal [20] proposed a modified Weibull geometric distribution which have monotonically increasing, decreasing, bathtub-shaped, and upside-down bathtub-shaped hazard rate functions. Saboor, Kamal and Ahmad [18] proposed a transmuted exponential Weibull distribution which have a bathtub-shaped and upside-down bathtub-shaped hazard rate functions. Kanchan, Neetu and Suresh Kumar [10] proposed Generalized Inverse Generalized Weibull (GIGW) distributions and discussed some mathematical properties of the distribution with a real life example. Inverse distributions, namely, Inverse Gamma, Inverse Generalized Gamma, Inverse Weibull [17], and Inverse Rayleigh [19], have also been studied in literature [6, 7, 9]. Khan and Jan [11, 12] worked on mixture models and discussed the stress-strength problem of the systems, where the strength follows finite mixture of two parameter Lindley distribution and stress follows exponential, Lindley distribution and mixture of two parameter Lindley distribution and obtained general expressions for the reliabilities of a system.

In this paper, we introduced a three parameter continuous distribution model, the Inverse Weibull Geometric Distribution (IWGD). It is the distribution of reciprocal of a variable distributed according to the generalized Weibull distribution. A comprehensive description of some mathematical properties of the IWGD are discussed with the hope that it will attract wider applications in reliability, engineering and other areas of research.

2. The Inverse Weibull Geometric Distribution (IWGD)

The Weibull geometric distribution with parameters $p \in (0,1), \alpha > 0 \; and \; \beta > 0$ is defined by its probability density function (pdf)

$$f(x; p, \alpha, \beta) = \alpha\beta^{\alpha}(1-p)x^{\alpha-1}e^{-(\beta x)^{\alpha}}\left\{1 - pe^{-(\beta x)^{\alpha}}\right\}^{-2}; \quad x > 0 \qquad (1)$$

The cumulative distribution function (cdf) of this distribution is

$$F(x) = \frac{1 - e^{-(\beta x)^{\alpha}}}{1 - pe^{-(\beta x)^{\alpha}}} \;\; ; \quad x > 0 \tag{2}$$

The inverse of Weibull geometric distribution called Inverse Weibull Geometric (IWGD) with parameters $p \in (0,1), \alpha > 0 \; and \; \beta > 0$ is defined by

$$f_{IWGD}(x;p,\alpha,\beta) = \alpha\beta^{\alpha}(1-p)x^{-(\alpha+1)}e^{-\left(\frac{\beta}{x}\right)^{\alpha}}\left\{1 - pe^{-\left(\frac{\beta}{x}\right)^{\alpha}}\right\}^{-2}$$

$$x,\alpha,\beta > 0 \quad and \;\; p\epsilon(0,1) \tag{3}$$

The cumulative distribution function of the distribution is given by

$$F_{IWGD}(x;p,\alpha,\beta) = \frac{(1-p)e^{-\left(\frac{\beta}{x}\right)^{\alpha}}}{1 - pe^{-\left(\frac{\beta}{x}\right)^{\alpha}}} \;\; ; \quad x,\alpha,\beta > 0 \quad and \;\; p\epsilon(0,1) \tag{4}$$

The Survival function and Hazard rate function of IWGD are

$$S_{IWG}(x) = \frac{1 - e^{-\left(\frac{\beta}{x}\right)^{\alpha}}}{1 - pe^{-\left(\frac{\beta}{x}\right)^{\alpha}}} \tag{5}$$

$$h_{IWGD}(x) = \frac{\alpha\beta^{\alpha}(1-p)x^{-(\alpha+1)}e^{-\left(\frac{\beta}{x}\right)^{\alpha}}}{\left(1 - e^{-\left(\frac{\beta}{x}\right)^{\alpha}}\right)\left(1 - pe^{-\left(\frac{\beta}{x}\right)^{\alpha}}\right)} \tag{6}$$

Special Cases

1) When For $\alpha = 1$, the new distribution is obtained called Inverse Exponential Geometric distribution (IEGD) with pdf and cdf respectively given as

$$f_{IEGD}(x;p,\beta) = \beta(1-p)x^{-2}e^{-\frac{\beta}{x}}\left\{1 - pe^{-\frac{\beta}{x}}\right\}^{-2} \; ; \; x,\beta > 0 \quad and \;\; p\epsilon(0,1)$$

$$F_{IEGD}(x;p,\beta) = \frac{(1-p)e^{-\frac{\beta}{x}}}{1 - pe^{-\frac{\beta}{x}}} \;\; ; \quad x,\beta > 0 \quad and \;\; p\epsilon(0,1)$$

2) When p approaches to zero, IWGD approaches to Inverse Weibull Distribution (IWD) with parameter $\alpha \; and \; \beta$.

3) When p approaches to zero and $\alpha = 2$, we get Inverse Rayleigh Distribution (IRD) with parameter β.
4) When p approaches to zero and $\alpha = 1$, we get Inverse Exponential Distribution (IED) with parameter β.

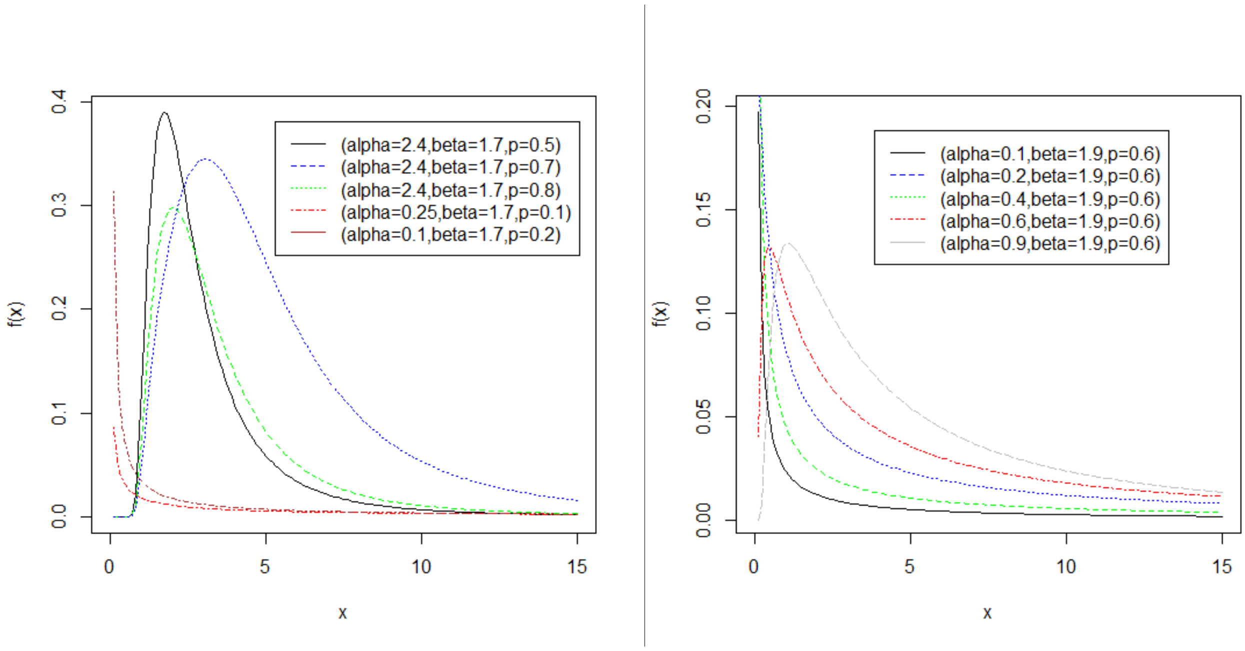

Fig. 1: Plots for IWGD function

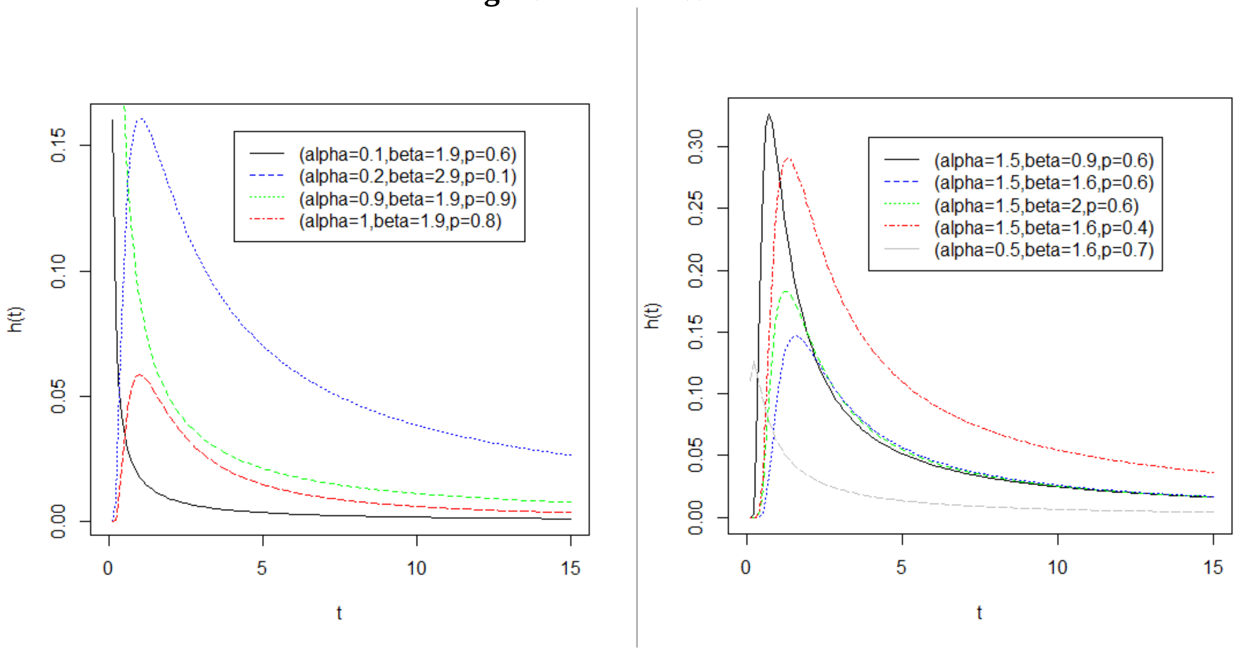

Fig. 2: Plots for Hazard Rate of IWGD

3. Moments of IWGD

The k^{th} order moment around zero for $X \sim IWG(\alpha, \beta, p)$ is written as

$$E(X^k) = \int_0^\infty x^k \alpha \beta^\alpha (1-p) x^{-(\alpha+1)} e^{-\left(\frac{\beta}{x}\right)^\alpha} \left\{1 - pe^{-\left(\frac{\beta}{x}\right)^\alpha}\right\}^{-2} dx$$

Substituting $\left(\frac{\beta}{x}\right)^\alpha = y$, we get

$$E(X^k) = (1-p)\beta^k \int_0^\infty y^{\frac{-k}{\alpha}} e^{-y} (1 - pe^{-y})^{-2} dy$$

$$E(X^k) = (1-p)\beta^k \Gamma\left(1 - \frac{k}{\alpha}\right) \sum_{j=0}^{\infty} p^j \, (j+1)^{\frac{k}{\alpha}}$$

For $\alpha = 1$, we get k^{th} order moment of Inverse Exponential distribution.

The moment generating function of IWGD of X can be written as

$$M(t) = (1-p) \sum_{k=0}^{\infty} \left[\frac{t^k}{k!} \beta^k \Gamma\left(1 - \frac{k}{\alpha}\right) \sum_{j=0}^{\infty} p^j \, (j+1)^{\frac{k}{\alpha}}\right], \quad for\ |t| < 1$$

The cumulative generating function of IWGD of X is given by

$$K(t) = logM(t) = log\left\{(1-p) \sum_{k=0}^{\infty} \left[\frac{t^k}{k!} \beta^k \Gamma\left(1 - \frac{k}{\alpha}\right) \sum_{j=0}^{\infty} p^j \, (j+1)^{\frac{k}{\alpha}}\right]\right\}, for\ |t| < 1$$

The mean and variance of IGWD are

$$E(X) = (1-p)\beta\Gamma\left(1 - \frac{1}{\alpha}\right) \sum_{j=0}^{\infty} p^j \, (j+1)^{\frac{1}{\alpha}}$$

$$V(X) = (1-p)\beta^2\Gamma\left(1 - \frac{2}{\alpha}\right) \sum_{j=0}^{\infty} p^j \, (j+1)^{\frac{2}{\alpha}} - \left[(1-p)\beta\Gamma\left(1 - \frac{1}{\alpha}\right) \sum_{j=0}^{\infty} p^j \, (j+1)^{\frac{1}{\alpha}}\right]^2$$

$$V(X) = (1-p)\beta^2 \left\{\Gamma\left(1 - \frac{2}{\alpha}\right) \sum_{j=0}^{\infty} p^j \, (j+1)^{\frac{2}{\alpha}} - (1-p)\left[\Gamma\left(1 - \frac{1}{\alpha}\right) \sum_{j=0}^{\infty} p^j \, (j+1)^{\frac{1}{\alpha}}\right]^2\right\}$$

3.1. Mixture of Two IWGD and Properties

Various mixture of distributions have been discussed in past by number of authors. Maclachlan and Krishnan [14], Everitt and Hand [8], Al-Hussaini and Sultan [3], Maclachlan and Peel [15]. They also discussed the properties of mixture distribution and have found them useful in many complex problems. The density function of mixture of two IWG distributions is called Mixed Inverse Weibull Geometric (MIWG) distribution and is given by

$$f_{MIWG}(x; \theta) = \lambda_1 f_1(x, \theta_1) + \lambda_2 f_2(x, \theta_2)$$

$$f_{MIWG}(x;\theta)=\sum_{k=1}^{2}\lambda_r f_r(x,\theta_r)$$

where,

$\theta_1=(p_1,\alpha_1,\beta_1)^T$ *and* $\theta_2=(p_2,\alpha_2,\beta_2)^T$, $f_r(x,\theta_r)$ corresponds to rth component of the mixture and is given by

$$f_r(x,\theta_r)=\alpha_r\beta_r^{\alpha}(1-p_r)x^{-(\alpha_r+1)}e^{-\left(\frac{\beta_r}{x}\right)^{\alpha_r}}\left\{1-p_r e^{-\left(\frac{\beta_r}{x}\right)^{\alpha_r}}\right\}^{-2}\ ;\ x,\alpha_r,\beta_r>0\ \ and\ p_r\epsilon(0,1)$$

and $\lambda_1+\lambda_2=1.$

Properties of MIGWD

The Survival function and Hazard rate function of MIWG distribution are

$$S_{MIWG}(x,\theta)=\sum_{r=1}^{2}\lambda_r\left(\frac{1-e^{-\left(\frac{\beta_r}{x}\right)^{\alpha_r}}}{1-p_r e^{-\left(\frac{\beta_r}{x}\right)^{\alpha_r}}}\right)$$

$$h_{MIWG}(x)=\sum_{k=1}^{2}\left(\lambda_r\alpha_r\beta_r^{\alpha}(1-p_r)x^{-(\alpha_r+1)}e^{-\left(\frac{\beta_r}{x}\right)^{\alpha_r}}\left\{1-p_r e^{-\left(\frac{\beta_r}{x}\right)^{\alpha_r}}\right\}^{-2}\right)$$
$$\times\left(\sum_{r=1}^{2}\lambda_r\left(\frac{1-e^{-\left(\frac{\beta_r}{x}\right)^{\alpha_r}}}{1-p_r e^{-\left(\frac{\beta_r}{x}\right)^{\alpha_r}}}\right)\right)^{-1}$$

The k^{th} order moment around zero for MIGWD can be written as

$$E(X^k)=\sum_{r=1}^{2}\int_0^{\infty}x^k\lambda_r\alpha_r\beta_r^{\alpha}(1-p_r)x^{-(\alpha_r+1)}e^{-\left(\frac{\beta_r}{x}\right)^{\alpha_r}}\left\{1-p_r e^{-\left(\frac{\beta_r}{x}\right)^{\alpha_r}}\right\}^{-2}dx$$

$$E(X^k)=\sum_{r=1}^{2}\lambda_r(1-p_r)\beta_r^k\Gamma\left(1-\frac{k}{\alpha_r}\right)\sum_{j=0}^{\infty}p_r^j\,(j+1)^{\frac{k}{\alpha_r}}$$

3.2. Estimation of Parameters

Let $x_1,x_2,\dots,x_n$ be a random sample from IGWD with unknown parameter vector $\phi=(p,\alpha,\beta)^T$. The log likelihood for $l=l(\phi;x)$ for ϕ is

$$l=n[log\alpha+\alpha log\beta+log(1-p)]-(\alpha+1)\sum_{i=1}^{n}logx_i-\sum_{i=1}^{n}\left(\frac{\beta}{x_i}\right)^{\alpha}$$
$$-2\sum_{i=1}^{n}log\left(1-pe^{\left(\frac{\beta}{x_i}\right)^{\alpha}}\right) \tag{7}$$

The score function $U(\phi) = \left(\frac{\partial l}{\partial p}, \frac{\partial l}{\partial \alpha}, \frac{\partial l}{\partial \beta}\right)^T$ has components

$$\frac{\partial l}{\partial p} = -n(1-p)^{-1} + 2\sum_{i=1}^{n} e^{\left(\frac{\beta}{x_i}\right)^\alpha}\left[1 - pe^{\left(\frac{\beta}{x_i}\right)^\alpha}\right]^{-1}$$

$$\frac{\partial l}{\partial \alpha} = n\alpha^{-1} + nlog\beta - \sum_{i=1}^{n} logx_i - \sum_{i=1}^{n}\left(\frac{\beta}{x_i}\right)^\alpha log\left(\frac{\beta}{x_i}\right)\left\{1 + 2pe^{\left(\frac{\beta}{x_i}\right)^\alpha}\left[1 - pe^{\left(\frac{\beta}{x_i}\right)^\alpha}\right]^{-1}\right\}$$

$$\frac{\partial l}{\partial \beta} = n\alpha\beta^{-1} - \alpha\beta^{\alpha-1}\sum_{i=1}^{n} x_i^{-\alpha}\left\{1 + 2pe^{\left(\frac{\beta}{x_i}\right)^\alpha}\left[1 - pe^{\left(\frac{\beta}{x_i}\right)^\alpha}\right]^{-1}\right\}$$

The maximum likelihood estimate (MLE) $\hat{\phi}$ of ϕ can be obtained by solving non-linear equations $U(\hat{\phi}) = 0$. These equations cannot be solved analytically but statistical software can be used to solve them numerically, for example, through the R-language or any iterative methods such as the NR (Newton-Raphson),BFGS (Broyden-Fletcher-Goldfarb-Shanno), BHHH (Berndt-Hall-Hall-Hausman), NM (Nelder-Mead), L-BFGS-B (Limited-Memory Quasi-Newton code for Bound-Constrained Optimization) and SANN (Simulated-Annealing).

Censored Case:

Censoring is the condition in which value of the observed value of some variables is unknown. In reliability studies particularly in survival analysis censoring occurs when the information about the survivals time of the observations under study is incomplete and therefore we are generally encountered with censored data. Let the independent random variables Z_i *and* C_i respectively denote the lifetime of the i^{th} individual and the censoring time and let $t_i = min(Z_i, C_i)$ for $i = 1,2,3, \dots, n$. The distribution of C_i does not depend on the unknown parameters of Z_i, where each Z_i follows the IGWD with parameters $\phi = (p, \alpha, \beta)^T$.

For censored case, while writing the likelihood function the data set splits into two parts one corresponds to censored data and another corresponds to those observations that are not censored. Let the sets of censored and uncensored observation respectively be denoted by C and F, and r the no of failures. The likelihood function for censored case can be written as

$$l = \prod_{i \in F} f(t_i) \times \prod_{i \in C} S(t_i)$$

where, $f(t_i)$ and $S(t_i)$ are the density function and survival function of the IGWD, respectively.

Hence from (3) and (4) the log likelihood for the IGWD can be written as

$$l = n[log\alpha + \alpha log\beta + log(1-p)] - (\alpha+1)\sum_{i\in F} logt_i - \sum_{i\in F}\left(\frac{\beta}{t_i}\right)^{\alpha} - 2\sum_{i\in F} log\left(1 - pe^{\left(\frac{\beta}{t_i}\right)^{\alpha}}\right)$$
$$+\sum_{i\in C} log\left(1 - e^{\left(\frac{\beta}{t_i}\right)^{\alpha}}\right) - \sum_{i\in C} log\left(1 - pe^{\left(\frac{\beta}{t_i}\right)^{\alpha}}\right)$$

The score function $U(\phi) = \left(\frac{\partial l}{\partial p}, \frac{\partial l}{\partial \alpha}, \frac{\partial l}{\partial \beta}\right)^T$ has components

$$\frac{\partial l}{\partial p} = -n(1-p)^{-1} + 2\sum_{i\in F} e^{\left(\frac{\beta}{t_i}\right)^{\alpha}}\left[1 - pe^{\left(\frac{\beta}{x_i}\right)^{\alpha}}\right]^{-1} + \sum_{i\in C} e^{\left(\frac{\beta}{t_i}\right)^{\alpha}}\left[1 - pe^{\left(\frac{\beta}{t_i}\right)^{\alpha}}\right]^{-1}$$

$$\frac{\partial l}{\partial \alpha} = n\alpha^{-1} + nlog\beta - \sum_{i\in F} logt_i - \sum_{i\in F} log\left(\frac{\beta}{t_i}\right)\left\{1 + 2pe^{\left(\frac{\beta}{t_i}\right)^{\alpha}}\left[1 - pe^{\left(\frac{\beta}{t_i}\right)^{\alpha}}\right]^{-1}\right\}\left(\frac{\beta}{t_i}\right)^{\alpha}$$
$$-\sum_{i\in C}\frac{(1-p)\left(\frac{\beta}{t_i}\right)^{\alpha} e^{\left(\frac{\beta}{t_i}\right)^{\alpha}} log\left(\frac{\beta}{t_i}\right)}{\left(1 - e^{\left(\frac{\beta}{t_i}\right)^{\alpha}}\right)\left(1 - pe^{\left(\frac{\beta}{t_i}\right)^{\alpha}}\right)}$$

$$\frac{\partial l}{\partial \beta} = n\alpha\beta^{-1} - \alpha\beta^{\alpha-1}\sum_{i\in F}\left\{1 + 2pe^{\left(\frac{\beta}{t_i}\right)^{\alpha}}\left[1 - pe^{\left(\frac{\beta}{t_i}\right)^{\alpha}}\right]^{-1}\right\} t_i^{-\alpha}$$
$$-\alpha\beta^{\alpha-1}\sum_{i\in C}\frac{(1-p)e^{\left(\frac{\beta}{t_i}\right)^{\alpha}} t_i^{-\alpha}}{\left(1 - e^{\left(\frac{\beta}{t_i}\right)^{\alpha}}\right)\left(1 - pe^{\left(\frac{\beta}{t_i}\right)^{\alpha}}\right)}$$

The maximum likelihood estimate (MLE) $\hat{\phi}$ of ϕ can be obtained by solving non-linear equations $U(\hat{\phi}) = 0$ using numerical methods.

4. Simulations and Applications

Estimation Based on simulations:

For checking the theoretical results, we simulate data by generating observations from IGWD for different sample sizes with number of repetitions 10,000 and values of parameters are chosen arbitrary. The values of the parameters are estimated using quasi- Newton method in R. the estimates of parameters with corresponding sample sizes are given in Table 1.

Table 1: Estimates of parameters for IGWD

n	α	β	p	$\hat{\alpha}$	$\hat{\beta}$	$\hat{p}$
50	1.1	0.02	0.2	1.2213	0.0045	0.9967
	1.2	0.02	0.3	1.4675	0.1046	0.9199
	1.3	0.02	0.4	1.1980	0.0061	0.9927
	1.4	0.03	0.5	1.5226	0.0408	0.9760
	1.4	0.04	0.6	1.5990	0.0945	0.9448
	1.4	0.05	0.7	1.4289	0.0584	0.9756
	1.5	0.06	0.8	1.6468	0.0473	0.9895
	1.6	0.07	0.8	1.4327	0.0534	0.9776
	1.7	0.08	0.8	1.6693	0.0404	0.9911
100	1.1	0.02	0.2	1.2942	0.0107	0.9944
	1.2	0.02	0.3	1.4743	0.0146	0.9949
	1.3	0.02	0.4	1.4067	0.0233	0.9928
	1.4	0.03	0.5	1.3589	0.0219	0.9871
	1.4	0.04	0.6	1.4601	0.0392	0.9862
	1.4	0.05	0.7	1.3314	0.0118	0.9946
	1.5	0.06	0.8	1.5888	0.0384	0.9838
	1.6	0.07	0.8	1.6331	0.0224	0.9956
	1.7	0.08	0.8	1.5073	0.0225	0.9922
250	1.1	0.02	0.2	1.3453	0.0152	0.9934
	1.2	0.02	0.3	1.3640	0.0150	0.9945
	1.3	0.02	0.4	1.3658	0.0103	0.9960
	1.4	0.03	0.5	1.4632	0.0165	0.9953
	1.4	0.04	0.6	1.4149	0.0173	0.9927
	1.4	0.05	0.7	1.4218	0.0347	0.9833
	1.5	0.06	0.8	1.4723	0.0250	0.9908
	1.6	0.07	0.8	1.4093	0.0140	0.9958
	1.7	0.08	0.8	1.4261	0.0145	0.9956
500	1.1	0.02	0.2	1.4530	0.0106	0.9972
	1.2	0.02	0.3	1.3850	0.0132	0.9952
	1.3	0.02	0.4	1.3946	0.0136	0.9955
	1.4	0.03	0.5	1.3319	0.0111	0.9948
	1.4	0.04	0.6	1.4373	0.0138	0.9960
	1.4	0.05	0.7	1.4242	0.0130	0.9962
	1.5	0.06	0.8	1.4857	0.0114	0.9972
	1.6	0.07	0.8	1.4481	0.0128	0.9967
	1.7	0.08	0.8	1.4729	0.0202	0.9931

4.1. Real Data Illustration

In this section we compare the results of fitting the IWGD, IWD, IEGD, IED and IRD to the data set studied by Meeker and Escobar [16], which gives the times of failure and running times for a sample of devices from an eld-tracking study of a larger system. At a certain point in time, 30 units were installed in normal service conditions. Two causes of failure were observed for each unit that failed: the failure caused by normal product wear and failure caused by an accumulation of randomly occurring damage from power-line voltage spikes during electric storms. The times are:
2.75, 0.13, 1.47, 0.23, 1.81, 0.30, 0.65, 0.10, 3.00, 1.73, 1.06, 3.00, 3.00, 2.12, 3.00, 3.00, 3.00, 0.02, 2.61, 2.93, 0.88, 2.47, 0.28, 1.43, 3.00, 0.23, 3.00, 0.80, 2.45, 2.66.

In order to compare the distribution models, we consider criteria like $-2\log(\mathrm{L})$, AIC (Akaike Information Criterion), BIC (Bayesian Information Criterion), CAIC (Consistant Akaike Information Criterion) and HQIC (Hannan-Quinn Information Criterion) for the data set. The better distribution corresponds to smaller $-2\mathrm{l}$, AIC, CAIC and HQIC.

The MLE for IWGD, IWD, IEGD, IED and IRD are given table 1 along with Akaike Information Criterion, Bayesian Information Criterion, Hannan-Quinn Information Criterion and Consistent Akaike Information Criterion for the data set.

Table 2 shows parameter MLEs to each one of the two fitted distributions for data set, values of $-2log(L)$, AIC, CAIC and HQIC. The values in Table 1 indicate that the IWGD model performs significantly better than its sub-models used here for fitting data set.

Table 2: The ML estimates, standard error, AIC, BIC and CAIC of the models based on data set

Model	-2LL	Estimates	St. Error	AIC	BIC	CAIC	HQIC
IWGD	325.1060	$\hat{\alpha} = 1.50414$	0.191013	331.1060	335.9388	331.7917	332.8098
		$\hat{\beta} = 1.02509$	0.718233				
		$\hat{p} = 0.98748$	0.015564				
IWD	335.5175	$\hat{\alpha} = 0.72031$	0.079020	339.5175	342.7393	339.8508	340.6533
		$\hat{\beta} = 11.59950$	2.301144				
IEGD	330.699	$\hat{\beta} = 3.08797$	1.362015	334.699	337.9208	335.0323	335.8348
		$\hat{p} = 0.84201$	0.091138				
IED	346.6261	$\hat{\beta} = 8.34602$	0.987354	348.6261	350.237	348.7342	349.194
IRD	507.971	$\hat{\beta} = 4.15026$	0.237946	509.971	511.5819	510.0791	510.5389

In the second example we considered the censored provided by David W. Hosmer and Stanley Lemeshow [5]. The survival times in months of 100 HIV patients are given below and * indicates that the data is censored.

5, 6*, 8, 3, 22, 1*, 7, 9, 3, 12, 2*, 12, 1, 15, 34, 1, 4, 19*, 3*, 2, 2*, 6, 60*, 7*, 60*,11, 2*, 5, 4*, 1*, 13, 3*, 2*, 1*, 30, 7*, 4*, 8*, 5*, 10, 2*, 9*, 36, 3*, 9*, 3*, 35, 8*, 1*, 5*, 11, 56*, 2*, 3*, 15, 1*, 10, 1*, 7*, 3*, 3*, 2*, 32, 3*, 10*, 11, 3*, 7*, 5*, 31, 5*, 58, 1*, 2*, 1, 3*, 43, 1*, 6*,53, 14, 4*, 54, 1*, 1*, 8*, 5*, 1*, 1*, 2*, 7*, 1*, 10, 24*, 7*, 12*, 4*, 57, 1*, 12*.

The MLE for IWGD, IWD, IEGD, IED and IRD are given table 3 along with Akaike Information Criterion, Bayesian Information Criterion, Hannan-Quinn Information Criterion and Consistent Akaike Information Criterion for the censored data.

Table 3: The ML estimates, standard error, AIC, BIC and CAIC of the models based on censored data

Model	-2LL	Estimates	St. Error	AIC	BIC	CAIC	HQIC
IWGD	104.1385	$\hat{\alpha} = 1.39976$	0.18206	110.1385	114.3421	110.9956	111.8423
		$\hat{\beta} = 0.02564$	0.01554				
		$\hat{p} = 0.99607$	0.00226				
IWD	120.5836	$\hat{\alpha} = 0.62515$	0.07612	124.5836	127.3860	124.9974	125.7194
		$\hat{\beta} = 0.53990$	0.16813				
IEGD	108.7143	$\hat{\beta} = 0.02658$	0.04016	112.7143	115.5167	113.1281	113.8501
		$\hat{p} = 0.97862$	0.03318				
IED	141.2617	$\hat{\beta} = 0.31223$	0.05700	143.2617	144.6629	143.395	143.8296
IRD	306.2023	$\hat{\beta} = 0.10481$	0.00956	308.2023	309.6035	308.3356	308.7702

Table 3 shows that values of $-2log(L)$, AIC, CAIC and HQIC are lowest for IWGD. So, we can conclude that IWGD model performs significantly better than its sub-models used here for fitting censored data set.

5. Conclusion

Here, we propose a new model, the so-called the Inverse Weibull Geometric Distribution which extends the Inverse Weibull distribution, Inverse Exponential Geometric Distribution, Inverse Exponential Distribution and Inverse Rayleigh Distribution in the analysis of data with real support. A pronounced reason for generalizing a standard distribution is because the generalized form extend larger flexibility in modelling real data. We derive expressions for the moments and for the moment generating function. The estimation of parameters is approached by the method of maximum likelihood. We consider the likelihood ratio statistic to compare the model with its baseline model. An application to real data show that the new distribution can be used adequately to provide better fits than the Inverse Weibull distribution, Inverse Exponential Geometric distribution, Inverse Exponential distribution and Inverse Rayleigh distribution.

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