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Article

A New Approach for Solving Bottleneck-Cost Transportation Problems

A.Ahmed[1], Afaq Ahmad[2] and J A Reshi[2]*

[1]*Department of Statistics and Operation Research, Aligarh Muslim University, Aligarh U.P*

[2]*Department of Statistics, University of Kashmir, Srinagar*

* Author to whom correspondence should be addressed; E-Mail: reshijavaid19@gmail.com

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Abstract: Transportation plays a vital role in financial development of the country. Mixture of transportation and mobility are directly involved with development of financial system of the country and for that mature transportation infrastructure is required. This paper analyzes bi-objective transportation problem and an alternative approach is proposed to solve the problem. It has been established that all the efficient pairs of values of the objective function can be attained at the extreme points of the feasible region. Based upon some related linear programming problems, a convergent algorithm is proposed to generate all the efficient pairs. The method is illustrated through a numerical example.

Keywords: Transportation Problem, Modified distribution (MODI) method, Bottleneck transportation problem, Efficient Solution, Bottleneck-Cost transportation problem.

Mathematical Subject Classifications (2010): 68M15, 62F15

1. Introduction

The classical transportation problem refers to a special class of linear programming problems. In a typical problem a product is to be transported from m sources to n destinations and their capacities are $a_1, a_2, ..., a_m$ and $b_1, b_2, ..., b_n$ resp. In addition there is a penalty c_{ij} associated with transporting unit of production from source i to destination j. This penalty may be cost or delivery time etc. A variable x_{ij} represents the unknown quantity to be shipped from source i to destination j. In general, the real life problems are modeled with multi-objectives which are measured in different scales. Bicriteria transportation problem arises when two objectives of conflicting nature are to be optimized. Since these objectives do not attain their optimum value at the same point, a single optimal solution is not available. Instead, a range of efficient solutions involving trade-offs between two objectives is obtained and the choice of solution is left to the decision maker. In normal situations the bicriteria transportation problem uses minimization of total cost as one of their objective while the other objectives may concern about delivery time, quantity of goods delivered, reliability of delivery, safety of delivery etc.

Bottleneck-cost Transportation Problem (BCTP) is a kind of a bicriteria transportation problem which had been proposed and also, solved by Aneja and Nair [9]. Isserman [6], Kassana [5] developed different approaches to generate the set of efficient solutions. The solution procedure of these methods depends on determining the set of efficient solutions and finally the decision maker is responsible for selecting the preferred solution out of this set. Yang and Gen [10] have proposed an approach called evolution program for bi-criteria transportation problem. Bodkhe et al. [2] used the fuzzy programming technique with hyperbolic member function to solve a bi-objective transportation problem as a vector minimum problem. Pandian and Anuradha [1] proposed dripping method for for finding a set of efficient solution to a bi-objective transportation problem.

In this paper we propose a new method for finding the set of efficient solutions to a bottleneck-cost transportation problem which is based on MODI method. The proposed method will provide the necessary and decision support to the decision maker while they are handling with time oriented logistic problems.

Mathematical Formulation:

(P) Minimize $z_1 = \sum_{i}^{m}\sum_{j}^{n} c_{ij} x_{ij}$

Minimize $z_2 = [\text{Maximize } t_{ij} / x_{ij} > 0]$

Subject to

$$\sum_{j}^{n} x_{ij} = a_i \quad , i = 1,2,3,...,m$$

$$\sum_{i}^{m} x_{ij} = b_j \quad , j = 1,2,3,..., n$$

$$x_{ij} \geq 0 \quad , i = 1,2,...,m ; j = 1,2,...,n$$

where a_i is the supply available at ith sourece; b_j is the demand required at jth destination; x_{ij} is the number of units shipped from ith source to jth destination; c_{ij} is the cost of transporting a unit from ith source to jth destination; t_{ij} is the time of transporting goods from ith source to jth destination; m is the number of sorces and n is the number of destinations.

Definitions:

Given two solution x and x^* of (P) in S, x^* is said to be dominate over x or (x^* is called a better solution of (P) than x) if any one of the following three conditions holds;

$z_1(x^*) < z_1(x)$ and $z_2(x^*) < z_2(x)$

$z_1(x^*) = z_1(x)$ and $z_2(x^*) < z_2(x)$

$z_1(x^*) < z_1(x)$ and $z_2(x^*) = z_2(x)$

If x^* is dominates over x, the corresponding pair of values $(z_1(x^*), z_2(x^*))$ is also said to dominate over the pair $(z_1(x), z_2(x))$.

Efficient Solution: A solution x^* of (P) is said to be an efficient solution if it is not dominated by any other solution of (P).

Efficient Pair: For an efficient solution x^* of (P), the pair of values $(z_1(x^*), z_2(x^*))$ is called an efficient pair.

Active Cost Transportation Problem: A cost transportation problem of BCTP is said to be active for any time M if the minimum time transportation corresponding to the cost transportation problem is M.

2. Algorithm of the Proposed Method

<u>Step1.</u> Construct two individual problems of the given BCTP namely, cost objective transportation problem (FOTP) and time objective transportation problem (SOTP).

<u>Step2.</u> First obtain the optimal solution of SOTP by the procedure given below:

(1) Obtain an initial basic feasible solution by the same method as for the normal transportation problem.

(2) Find T_k corresponding to the current feasible solution, where T_k is the maximum time associated with k^{th} feasible plan.

(3) Cross out all the non-basic cells for which $t_{ij} \geq T_k$.

(4) Draw a closed path for the basic cell associated with T_k such that when the values at the corner elements are shifted, this variable reduces to zero and no other variable becomes negative. If no closed path can be traced out then solution is optimal, let it be T_O, otherwise repeat 2.2.

<u>Step3.</u> Now obtain the optimal solution of FOTP by MODI method and using this solution in SOTP, find the corresponding time transportation. Let it be T_C.

<u>Step4.</u> For each time M in [T_O , T_C], construct the active cost transportation problem and solve it by MODI method, an optimal solution to the cost transportation problem, x is obtained. Then the vector (X, M) is an efficient solution to BCTP.

Numerical example:

Table 1: Consider the foloowing Transportation Problem

	D_1	D_2	D_3	D_4	Supply
O_1	5 10	2 68	4 73	3 52	8
O_2	6 66	4 95	9 30	5 21	19
O_3	2 97	3 63	8 19	1 23	17
Demand	11	3	14	16	

The lower right corner in each cell is the time of transportation on the corresponding route and the upper left corner cell is the cost per unit transportation on that route.

Now, the SOTP of BCTP is given below:

	D_1	D_2	D_3	D_4	Supply
O_1	10	68	73	52	8
O_2	66	95	30	21	19
O_3	97	63	19	23	17
Demand	11	3	14	16	

Using North West Corner rule, the initial basic feasible solution is $x_{11} = 8$, $x_{21} = 3$, $x_{22} = 3$, $x_{23} = 13$, $x_{33} = 1$, $x_{34} = 16$.

The shipment times are $t_{11} = 10$, $t_{21} = 66$, $t_{22} = 95$, $t_{23} = 30$, $t_{33} = 19$, $t_{34} = 23$.

Now $T_1 = \max\{10, 66, 95, 30, 19, 23\} = 95$ belongs to cell (2, 2). Thus the cell (3, 1) is crossed out, and we make a closed path for the cell (2, 2) as shown below:

	D_1	D_2	D_3	D_4	Supply
O_1	10 (8)	68	73	52	8
O_2	66 (3)	95 – (3)	30 + (13)	21	19
O_3		63 +	19 – (1)	23 (16)	17
Demand	11	3	14	16	

It is clear from the closed path that x_{22} can be decreased by only one unit otherwise cell (3, 3) will become negative. The new solution is given below:

	D_1	D_2	D_3	D_4	Supply
O_1	10 (8)	68	73	52	8
O_2	66 (3)	95 (2)	30 (14)	21	19
O_3		63 (1)	19	23 (16)	17
Demand	11	3	14	16	

Thus T_2 = max {10, 66, 95, 30, 63, 23}

= 95 belong to cell (2, 2).

Now after drawing a closed path for the cell (2, 2), the new solution is given below:

	D_1	D_2	D_3	D_4	Supply
O_1	10 (6)	68 (2)	73	52	8
O_2	66 (5)	95	30 (14)	21	19
O_3		63 (1)	19	23 (16)	17
Demand	11	3	14	16	

Here T_3 = max {10, 68, 66, 30, 63, 23}

= 68 belongs to cell (1, 2)

Since t_{22} and t_{13} are greater than t_{12} hence, we cross it and drawing a closed path for the cell (1, 2), the new solution is given below:

	D_1	D_2	D_3	D_4	Supply
O_1	10 (6)	68		52 (2)	8
O_2	66 (5)		30 (14)	21	19
O_3		63 (3)	19	23 (14)	17
Demand	11	3	14	16	

Here T_4 = max {10, 52, 66, 30, 63, 23 = 66 belongs to cell (2, 1)

Since $t_{12} > T_4$, hence cross it. After crossing it we can't make a closed path for the cell (2, 1). Thus T_4 is optimal solution.

Now, the cost transportation table of BCTP is given below:

	D_1	D_2	D_3	D_4	Supply
O_1	5	2	4	3	8
O_2	6	4	9	5	19
O_3	2	3	8	1	17
Demand	11	3	14	16	

Using MODI method, the optimal solution is x_{13} = 8,x_{21} = 10, x_{22} = 3, x_{23} = 6, x_{31} = 1, x_{34} = 16 with minimum transportation cost 176 and using this solution in SOTP the minimum time transportation is 97.

Now we have *To* = 66, *Tm* = 97 and the time *M* = {66, 68, 73, 95, 97}

Now, the active cost transportation problem of BCTP for *M* = 66 is given below:

	D_1	D_2	D_3	D_4	Supply
O_1	5			3	8
O_2	6		9	5	19
O_3		3	8	1	17
Demand	11	3	14	16	

Using MODI method, the optimal solution is x_{11} = 6,x_{14} = 2, x_{21} = 5, x_{23} = 14, x_{32} = 3, x_{34} =14 with minimum transportation cost 215.

Now, the active cost transportation problem of BCTP for *M* = 68 is given below:

	D_1	D_2	D_3	D_4	Supply
O_1	5	2		3	8
O_2	6		9	5	19
O_3		3	8	1	17
Demand	11	3	14	16	

Using MODI method, the optimal solution is x_{11} = 5,x_{12} = 3, x_{21} = 6, x_{23} = 13, x_{33} = 1, x_{34} =16 with minimum transportation cost 208

Now, the active cost transportation problem of BCTP for *M* = 73 is given below:

	D_1	D_2	D_3	D_4	Supply
O_1	5	2	4	3	8
O_2	6		9	5	19
O_3		3	8	1	17
Demand	11	3	14	16	

Using MODI method, the optimal solution is $x_{12} = 2, x_{13} = 6$, $x_{21} = 11$, $x_{23} = 8$, $x_{32} = 1$, $x_{34} = 16$ with minimum transportation cost 185.

Now, the active cost transportation problem of BCTP for $M = 95$ is given below:

	D_1	D_2	D_3	D_4	Supply
O_1	5	2	4	3	8
O_2	6	4	9	5	19
O_3		3	8	1	17
Demand	11	3	14	16	

Using MODI method, the optimal solution is $x_{13} = 8, x_{21} = 11$, $x_{22} = 3$, $x_{23} = 5$, $x_{33} = 1$, $x_{34} = 16$ with minimum transportation cost 179.

Now, the efficient solutions to the BCTP are given below:

S.No.	Efficient solution of BCTP	Objective value of BCTP
1	$x_{11} = 6, x_{14} = 2$, $x_{21} = 5$, $x_{23} = 14$, $x_{32} = 3$, $x_{34} = 14$	(215, 66)
2	$x_{11} = 5, x_{12} = 3$, $x_{21} = 6$, $x_{23} = 13$, $x_{33} = 1$, $x_{34} = 16$	(208, 68)
3	$x_{12} = 2, x_{13} = 6$, $x_{21} = 11$, $x_{23} = 8$, $x_{32} = 1$, $x_{34} = 16$	(185,73)
4	$x_{13} = 8, x_{21} = 11$, $x_{22} = 3$, $x_{23} = 5$, $x_{33} = 1$, $x_{34} = 16$	(179,95)
5	$x_{13} = 8, x_{21} = 10$, $x_{22} = 3$, $x_{23} = 6$, $x_{31} = 1$, $x_{34} = 16$	(176, 97)

4. Conclusion

In this paper, the proposed method provides the set of efficient solutions for bi-objective transportation problems. The proposed method is quite simple from the computational point of view and also, easy to understand and apply. This method provides a set of transportation schedules to BCTP which helps the decision makers to select an appropriate solution, depending on their financial position and the extent of bottleneck that they can afford. The future research work may be considered to introduce the mathematical formulation of the proposed method and algorithm.

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