Generating Exact Solutions of Two-Dimensional Incompressible Navier-Stokes Equations

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Abstract: This paper investigates exact solutions of two dimensional incompressible Navier-Stokes equations (2D NSEs) for a time dependent pressure gradient term using Orlowski and Sobczyk transformation (OST) and Cole-Hopf transformation (CHT). Separation of variables method is applied to find the solutions of 2D heat equation.

Keywords: Burgers equation, Cole-Hopf transformation (CHT), Orlowski and Sobczyk transformation (OST).

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1. Introduction

The Navier–Stokes equations are important governing equations in the fluid dynamics which describe the motion of fluid. These equations arise from applying Newton’s second law to fluid motion, together with the assumption that the fluid feels forces due to pressure, viscosity and perhaps an external force. They are useful because they describe the physics of many things of academic and economic interest.
However, NSEs are nonlinear in nature and it is difficult to solve these equations analytically. Despite the concentrated research on Navier Stokes equations, their universal solutions are not achieved. The full solutions of the three-dimensional NSEs remain one of the open problems in mathematical physics. The exact solutions for the NSEs can be obtained are of particular cases. Exact solutions on the other hand are very important for many reasons. They provide reference solutions to verify the accuracies of many approximate methods. In order to understand the non-linear phenomenon of NSE, one needs to study 2D NSEs. Since these equations posses’ diffusion, advection and pressure gradient terms of the full 3D NSEs, these incorporate all the main mathematical features of the full 3D NSEs.

Applying OST we have reduced 2D NSEs to 2D viscous Burgers’ equations. These equations are mathematical model which are widely used for various physical applications, such as modeling of gas dynamics and traffic flow, shock waves in examining the chemical reaction-diffusion model of Brusselator etc. So a number of analytical and numerical studies on NSEs as well as Burgers’ equations have been conducted to find the exact solutions of the governing equations.


from general analytical solutions via CHT. Lei Zhang, Lisha Wang, Xiaohua Ding [15] developed exact finite-difference schemes for the 2D nonlinear coupled viscous Burgers’ equations using the analytical solutions. Mohammad Tamsir, Vineet Kumar Srivastava [16] used a semi-implicit finite difference approach to find the numerical solutions of the two-dimensional coupled Burgers’ equations. T.X. Yan, L.S. Yue [17] presented variable separable solutions for the (2+1)–dimensional equation.

In this paper, we reduce 2D NSEs into 2D Burgers equations by applying OST. Then after applying CHT 2D Burgers’ equations will be reduced to 2D diffusion equation. By using separation of variables method we will solve diffusion equation. Then applying CHT and inverse OST we get the exact solutions of 2D NSEs.

2. Mathematical Formulation

The dimensionalised governing equations of the fluid flow are given respectively by the continuity equation

\[ \frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0 \]

x-momentum equation

\[ \frac{\partial u^*}{\partial t^*} + u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = -\frac{1}{\rho} \frac{\partial p^*}{\partial x^*} + \nu \left( \frac{\partial^2 u^*}{\partial x^*^2} + \frac{\partial^2 u^*}{\partial y^*^2} \right) \]

y-momentum equation

\[ \frac{\partial v^*}{\partial t^*} + u^* \frac{\partial v^*}{\partial x^*} + v^* \frac{\partial v^*}{\partial y^*} = -\frac{1}{\rho} \frac{\partial p^*}{\partial y^*} + \nu \left( \frac{\partial^2 v^*}{\partial x^*^2} + \frac{\partial^2 v^*}{\partial y^*^2} \right) \]

where \( u \) and \( v \) are the velocity components in the \( x \) and \( y \) directions respectively, \( p \) is the pressure, \( \rho \) is the constant density and \( \nu \) is the kinematic viscosity.

Using the dimensionless definitions in [9],

\[ t = \frac{t \ast U}{h} \quad x = \frac{x \ast h}{U} \quad y = \frac{y \ast h}{U} \quad u = \frac{u \ast U}{h} \quad v = \frac{v \ast U}{h} \quad p = \frac{p \ast p \ast U^2}{h^2} \]

The dimensionalised governing equations are then converted into the non-dimensional form 2D Navier-Stokes equations (2D NSEs) as

\[ u_x + uu_x + vu_y = -p_x + \frac{1}{\text{Re}} (u_{xx} + u_{yy}) \quad (1) \]

\[ v_x + uv_x + vv_y = -p_y + \frac{1}{\text{Re}} (v_{xx} + v_{yy}) \quad (2) \]

\[ u_x + v_y = 0 \quad (3) \]
where \( \frac{1}{Re} = \frac{\nu}{U \ell} \); \( p_x = f(t) \); \( p_y = g(t) \).

As a result equation (1) can be rewritten as

\[ u_x + uu_x + vu_y = f(t) + \frac{1}{Re} (u_{xx} + u_{yy}) \quad (4) \]

“(2)” can be written as

\[ v_x + uv_x + vv_y = g(t) + \frac{1}{Re} (u_{xx} + u_{yy}) \quad (5) \]

3. Generating Exact Solutions of 2D NSEs

We apply OST [18] as

\[ x' = x - \phi(t), \quad y' = y - \psi(t), \quad t' = t, \quad u'(x', y', t') = u(x, y, t) - W(t), \quad v' = v(x, y, t) - W'(t) \quad (6) \]

With \( W(t) = \int_0^t f(\tau) d\tau, \phi(t) = \int_0^t W(\tau) d\tau \quad (7) \)

And \( W'(t) = \int_0^t f'(\tau) d\tau, \psi(t) = \int_0^t W'(\tau) d\tau \quad (8) \)

Here \( W(t) = \int_0^t f(\tau) d\tau \)

\[ = [F(\tau)]_0^t \]

\[ = F(t) - F(0) \]

Again \( \phi(t) = \int_0^t W(\tau) d\tau \)

\[ = \int_0^t (F(\tau) - F(0)) d\tau \]

\[ = [G(\tau) - \tau F(0)]_0^t \]

\[ = G(t) - t F(0) - G(0) \]

Now \( W'(t) = \int_0^t f'(\tau) d\tau \)

\[ = [F'(\tau)]_0^t \]

\[ = F'(t) - F'(0) \]

Again \( \psi(t) = \int_0^t W'(\tau) d\tau \)

\[ = \int_0^t (F'(\tau) - F'(0)) d\tau \]

\[ = [G'(\tau) - \tau F'(0)]_0^t \]
\[ G'(t) - tF'(0) - G'(0) \]

Thus we get
\[ x' = x - G(t) + tF(0) + G(0) \]  \hspace{1cm} (9)
\[ y' = y - G'(t) + tF'(0) + G'(0) \]  \hspace{1cm} (10)
\[ t' = t \]  \hspace{1cm} (11)
\[ u'(x', y', t') = u(x, y, t) - F(t) + F(0) \]  \hspace{1cm} (12)
\[ v'(x', y', t') = v(x, y, t) - F'(t) + F'(0) \]  \hspace{1cm} (13)

Now
\[ \frac{\partial u}{\partial t} = \frac{\partial}{\partial t} (u' + F(t) - F(0)) \]
\[ = \frac{\partial u'}{\partial t} + f(t) \]
\[ = \frac{\partial u'}{\partial x'} \frac{\partial x'}{\partial t} + \frac{\partial u'}{\partial y'} \frac{\partial y'}{\partial t} + \frac{\partial u'}{\partial t'} \frac{\partial t'}{\partial t} + f(t) \]
\[ = \frac{\partial u'}{\partial x'} \frac{\partial}{\partial t} (x - G(t) + tF(0) + G(0)) + \frac{\partial u'}{\partial y'} \frac{\partial}{\partial t} (y - G'(t) + tF'(0) + G'(0)) + \frac{\partial u'}{\partial t'} \frac{\partial}{\partial t} + f(t) \]
\[ = \frac{\partial u'}{\partial x'} (-F(t) + F(0)) + \frac{\partial u'}{\partial y'} (-F'(t) + F'(0)) + \frac{\partial u'}{\partial t'} + f(t). \]

\[ \frac{\partial u}{\partial x} = \frac{\partial}{\partial x} (u' + F(t) - F(0)) \]
\[ = \frac{\partial u'}{\partial x} \]
\[ = \frac{\partial u'}{\partial x'} \frac{\partial x'}{\partial x} + \frac{\partial u'}{\partial y'} \frac{\partial y'}{\partial x} + \frac{\partial u'}{\partial t'} \frac{\partial t'}{\partial x} \]
\[ = \frac{\partial u'}{\partial x'} \frac{\partial}{\partial x} (x - G(t) + tF(0) + G(0)) + \frac{\partial u'}{\partial x'} \frac{\partial}{\partial x} (y - G'(t) + tF'(0) + G'(0)) + \frac{\partial u'}{\partial t'} \frac{\partial}{\partial x} \]
\[ = \frac{\partial u'}{\partial x'} \]

\[ \frac{\partial u}{\partial y} = \frac{\partial}{\partial y} (u' + F(t) - F(0)) \]
\[ = \frac{\partial u'}{\partial y} \]
\[ = \frac{\partial u'}{\partial x'} \frac{\partial x'}{\partial y} + \frac{\partial u'}{\partial y'} \frac{\partial y'}{\partial y} + \frac{\partial u'}{\partial t'} \frac{\partial t'}{\partial y} \]
\[ = \frac{\partial u'}{\partial x'} \frac{\partial}{\partial y} (x - G(t) + tF(0) + G(0)) + \frac{\partial u'}{\partial y'} \frac{\partial}{\partial y} (y - G'(t) + tF'(0) + G'(0)) + \frac{\partial u'}{\partial t'} \frac{\partial}{\partial y} \]
\[ v' = \frac{\partial u'}{\partial y}. \]

Also
\[ \frac{\partial^2 u}{\partial x^2} = \frac{\partial}{\partial x} \left( \frac{\partial u}{\partial x} \right) \]
\[ = \frac{\partial}{\partial x} \left( \frac{\partial u'}{\partial x} \right) \]
\[ = \frac{\partial}{\partial x} \left( \frac{\partial u'}{\partial x'} \right) \cdot \frac{\partial x'}{\partial x} + \frac{\partial}{\partial y} \left( \frac{\partial u'}{\partial x} \right) \cdot \frac{\partial y'}{\partial y} + \frac{\partial}{\partial t} \left( \frac{\partial u'}{\partial x} \right) \cdot \frac{\partial t'}{\partial t} \]
\[ = \frac{\partial^2 u'}{\partial x^2} \cdot \frac{\partial}{\partial x} (x - G(t) + tF(0) + G(0)) + \frac{\partial^2 u'}{\partial y^2} \cdot \frac{\partial}{\partial y} (y - G'(t) + tF'(0) + G'(0)) + \frac{\partial^2 u'}{\partial t^2} \cdot \frac{\partial}{\partial t} (\frac{\partial u'}{\partial t' \partial x} \cdot \frac{\partial t}{\partial x}) \]
\[ = \frac{\partial^2 u'}{\partial x^2}. \]

\[ \frac{\partial^2 u}{\partial y^2} = \frac{\partial}{\partial y} \left( \frac{\partial u}{\partial y} \right) \]
\[ = \frac{\partial}{\partial y} \left( \frac{\partial u'}{\partial y} \right) \]
\[ = \frac{\partial}{\partial x} \left( \frac{\partial u'}{\partial y} \right) \cdot \frac{\partial x'}{\partial y} + \frac{\partial}{\partial y} \left( \frac{\partial u'}{\partial y} \right) \cdot \frac{\partial y'}{\partial y} + \frac{\partial}{\partial t} \left( \frac{\partial u'}{\partial y} \right) \cdot \frac{\partial t'}{\partial t} \]
\[ = \frac{\partial^2 u'}{\partial x^2} \cdot \frac{\partial}{\partial y} (x - G(t) + tF(0) + G(0)) + \frac{\partial^2 u'}{\partial y^2} \cdot \frac{\partial}{\partial y} (y - G'(t) + tF'(0) + G'(0)) + \frac{\partial}{\partial t} \left( \frac{\partial u'}{\partial t' \partial y} \cdot \frac{\partial t}{\partial x} \right) \]
\[ = \frac{\partial^2 u'}{\partial y^2}. \]

\[ \frac{\partial v}{\partial t} = \frac{\partial}{\partial t} (v' + F'(t) - F'(0)) \]
\[ = \frac{\partial v'}{\partial t} + f'(t) \]
\[ = \frac{\partial v}{\partial x} \cdot \frac{\partial x'}{\partial t} + \frac{\partial v}{\partial y} \cdot \frac{\partial y'}{\partial t} + \frac{\partial v}{\partial t} \cdot \frac{\partial t'}{\partial t} + f'(t) \]
\[ = \frac{\partial v'}{\partial x} \cdot \frac{\partial}{\partial t} (x - G(t) + tF(0) + G(0)) + \frac{\partial v}{\partial y} \cdot \frac{\partial y'}{\partial t} (y - G'(t) + tF'(0) + G'(0)) + \frac{\partial v'}{\partial t} \cdot \frac{\partial t}{\partial t} + f'(t) \]
\[ = \frac{\partial v'}{\partial x} \cdot (-F(t) + F(0)) + \frac{\partial v}{\partial y} \cdot (-F'(t) + F'(0)) + \frac{\partial v'}{\partial t} + f'(t) \]
\[
\frac{\partial v}{\partial x} = \frac{\partial}{\partial x} \left( v' + F'(t) + F'(0) \right) \\
= \frac{\partial v'}{\partial x} \\
= \frac{\partial v'}{\partial x} \cdot \frac{\partial x}{\partial x} + \frac{\partial v'}{\partial y} \cdot \frac{\partial y}{\partial x} + \frac{\partial v'}{\partial t} \cdot \frac{\partial t}{\partial x} \\
= \frac{\partial v'}{\partial x} \cdot \frac{\partial}{\partial x} \left( x - G(t) + tF(0) + G(0) \right) + \frac{\partial v'}{\partial y} \cdot \frac{\partial}{\partial x} \left( y - G'(t) + tF'(0) + G'(0) \right) + \frac{\partial v'}{\partial t} \cdot \frac{\partial}{\partial x} \\
= \frac{\partial v'}{\partial x}' \\
\frac{\partial v}{\partial y} = \frac{\partial}{\partial y} \left( v' + F'(t) + F'(0) \right) \\
= \frac{\partial v'}{\partial y} \\
= \frac{\partial v'}{\partial y} \cdot \frac{\partial y}{\partial y} + \frac{\partial v'}{\partial x} \cdot \frac{\partial x}{\partial y} + \frac{\partial v'}{\partial t} \cdot \frac{\partial t}{\partial y} \\
= \frac{\partial v'}{\partial y} \cdot \frac{\partial}{\partial y} \left( x - G(t) + tF(0) + G(0) \right) + \frac{\partial v'}{\partial x} \cdot \frac{\partial}{\partial y} \left( y - G'(t) + tF'(0) + G'(0) \right) + \frac{\partial v'}{\partial t} \cdot \frac{\partial}{\partial y} \\
= \frac{\partial v'}{\partial y}' \\
\text{Also} \quad \frac{\partial^2 v}{\partial x^2} = \frac{\partial}{\partial x} \left( \frac{\partial v}{\partial x} \right) \\
= \frac{\partial}{\partial x} \left( \frac{\partial v'}{\partial x} \right) \\
= \frac{\partial}{\partial x} \left( \frac{\partial v'}{\partial y} \cdot \frac{\partial y}{\partial x} + \frac{\partial v'}{\partial x} \cdot \frac{\partial x}{\partial x} + \frac{\partial v'}{\partial t} \cdot \frac{\partial t}{\partial x} \right) \\
= \frac{\partial^2 v'}{\partial x^2} \cdot \frac{\partial}{\partial x} \left( x - G(t) + tF(0) + G(0) \right) + \frac{\partial^2 v'}{\partial y \partial x} \cdot \frac{\partial}{\partial x} \left( y - G'(t) + tF'(0) + G'(0) \right) + \frac{\partial v'}{\partial t} \cdot \frac{\partial}{\partial x} \\
= \frac{\partial^2 v'}{\partial x^2} \cdot 0 + \frac{\partial^2 v'}{\partial y \partial x} \cdot 0 + \frac{\partial}{\partial t} \left( \frac{\partial v'}{\partial x} \right) \cdot 0 \\
= \frac{\partial^2 v'}{\partial x^2} \\
\frac{\partial^2 v}{\partial y^2} = \frac{\partial}{\partial y} \left( \frac{\partial v}{\partial y} \right)
y} \frac{\partial v'}{\partial y'} + \frac{\partial^2 v'}{\partial y'^2} \cdot 0 \\
= \frac{\partial^2 v'}{\partial y'^2}.

Substituting the transformed derivatives in “(2)” , we get

\begin{align*}
&u'_x (-F(t) + F(0)) + u'_y (-F'(t) + F'(0)) + u'_x + f(t) + (u' + F(t) - F(0))u'_x + (v' + F'(t) - F'(0))v'_y = f(t) + \frac{1}{Re} (u'_{xx} + u'_{yy}) \\
\Rightarrow &u'_x + u'u'_x + v'u'_y = \frac{1}{Re} (u'_{xx} + u'_{yy}) \\
\text{(14)}
\end{align*}

Similarly, substituting the transformed derivatives in “(3)” , we get

\begin{align*}
&v'_x + u'v'_x + v'v'_y = \frac{1}{Re} (v'_{xx} + v'_{yy}) \\
\text{(15)}
\end{align*}

From “ (4)” , we obtain

\begin{align*}
u'_x + v'_y = 0 \\
\text{(16)}
\end{align*}

Again, we know that the non-dimensional form of 2D Burgers equations are

\begin{align*}
&\frac{\partial u^*}{\partial t^*} + u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = \frac{1}{Re} \left( \frac{\partial^2 u^*}{\partial x^{**}^2} + \frac{\partial^2 u^*}{\partial y^{**}^2} \right) \\
\text{(17)}
\end{align*}

\begin{align*}
&\frac{\partial v^*}{\partial t^*} + u^* \frac{\partial v^*}{\partial x^*} + v^* \frac{\partial v^*}{\partial y^*} = \frac{1}{Re} \left( \frac{\partial^2 v^*}{\partial x^{**}^2} + \frac{\partial^2 v^*}{\partial y^{**}^2} \right) \\
\text{(18)}
\end{align*}

So, “ (14)” , “ (15)”are the transformed 2D Navier-Stokes equations after applying OST and these are analogous to non-dimensional form of 2D Burgers equation “ (17)” , “ (18)”.

Now we need to solve “(14)” , “(15)” with initial conditions

\begin{align*}
u'(x', y', 0) = u_0(x', y'), v'(x', y', 0) = v_0'(x', y') \text{where } -\infty < x' < \infty, -\infty < y' < \infty.
\end{align*}

“(14)” and “(15)” can be linearized by the Cole-Hopf transformation [11],[13],[14],[19]
\[ u'(x', y', t') = -\frac{2 \phi_i'}{\text{Re} \phi'} \]  
(19)

\[ v'(x', y', t') = -\frac{2 \phi_j'}{\text{Re} \phi'} \]  
(20)

By using these transformations 2D coupled Burgers’ equations can be reduced to 2D diffusion equation

\[ \psi_{',r} = \frac{1}{\text{Re}} \left( \psi_{',x'} + \psi_{',y'} \right) \]  
(21)

which is the well-known second order PDE called heat or diffusion equation [11], [13],[14],[19].

Consider a general solution of “(21)” of the form similar analogy to [11],[14].

\[ \psi'(x', y', t') = a_1 + a_2 x' + a_3 y' + a_4 x'y' + X(x')Y(y')T(t') \]  
(22)

which is the sum of the solution \( \psi_1'(x', y', t') = a_1 + a_2 x' + a_3 y' + a_4 x'y' \) and the separable solution \( \psi_2'(x', y', t') = X(x')Y(y')T(t') \) where \( a_1,a_2,a_3,a_4 \) are arbitrary constants and \( X,Y,T \) are functions of \( x',y',t' \) respectively. Then “(21)” becomes

\[ XYT' = \frac{1}{\text{Re}} \left( X''YT + XY''T \right) \]

Dividing by \( XYT/\text{Re} \) on both sides, we get

\[ \frac{\text{Re}T'}{T'} = \frac{X''}{X} + \frac{Y''}{Y} \]

Since the left side is a function of \( t' \) alone, while the right side is a function of \( x' \) and \( y' \), we see that each side must be a constant, say \( -\lambda^2 \) (which is needed for boundedness). Thus

\[ T' + \frac{\lambda^2 T'}{\text{Re}} = 0 \]  
(23)

\[ \frac{X''}{X} + \frac{Y''}{Y} = -\lambda^2 \]  
(24)

“(24)” can be written as

\[ \frac{X''}{X} = -\frac{Y''}{Y} - \lambda^2 \]

And since the left side depends only on \( x' \) while the right side depends only on \( y' \) each side must be a constant, say \( -\mu^2 \). Thus
\[ X'' + \mu^2 X = 0 \]  \hspace{1cm} (25)
\[ Y'' + (\lambda^2 - \mu^2)Y = 0 \]  \hspace{1cm} (26)

Solutions to “(25)”, “(26)” and “(23)” are given by

\[ X = b_1 \cos \mu x' + c_1 \sin \mu x', \quad Y = b_2 \cos \sqrt{\lambda^2 - \mu^2} y' + c_2 \sin \sqrt{\lambda^2 - \mu^2} y', \quad T = b_3 e^{-\frac{\lambda y'}{Re}} \]  \hspace{1cm} (27)

It follows that a solution to “(21)” is given by

\[ \psi(x', y', t') = (b_1 \cos \mu x' + c_1 \sin \mu x') (b_2 \cos \sqrt{\lambda^2 - \mu^2} y' + c_2 \sin \sqrt{\lambda^2 - \mu^2} y') \left( b_3 e^{-\frac{\lambda y'}{Re}} \right) \]  \hspace{1cm} (28)

Thus the general solution of 2D diffusion equation is given by

\[ \psi(x', y', t') = a_1 + a_2 x' + a_3 y' + a_4 x' y' + \left( b_1 \cos \mu x' + c_1 \sin \mu x' \right) \left( b_2 \cos \sqrt{\lambda^2 - \mu^2} y' + c_2 \sin \sqrt{\lambda^2 - \mu^2} y' \right) \left( b_3 e^{-\frac{\lambda y'}{Re}} \right) \]  \hspace{1cm} (29)

By using CHT we obtain the general solution of 2D viscous Burgers equation as

\[ u(x', y', t') = \frac{2}{Re} \left( \frac{a_2 + a_3 y' + (-b_1 \mu \sin \mu x' + c_1 \mu \cos \mu x') \left( b_2 \cos \sqrt{\lambda^2 - \mu^2} y' + c_2 \sin \sqrt{\lambda^2 - \mu^2} y' \right) \left( b_3 e^{-\frac{\lambda y'}{Re}} \right)}{a_1 + a_2 x' + a_3 y' + a_4 x' y' + \left( b_1 \cos \mu x' + c_1 \sin \mu x' \right) \left( b_2 \cos \sqrt{\lambda^2 - \mu^2} y' + c_2 \sin \sqrt{\lambda^2 - \mu^2} y' \right) \left( b_3 e^{-\frac{\lambda y'}{Re}} \right)} \right) \]  \hspace{1cm} (30)

\[ v(x', y', t') = \frac{2}{Re} \left( \frac{a_3 + a_4 y' + (b_1 \cos \mu x' + c_1 \sin \mu x') \left( -b_2 \sin \sqrt{\lambda^2 - \mu^2} y' + c_2 \cos \sqrt{\lambda^2 - \mu^2} y' \right) \left( \sqrt{\lambda^2 - \mu^2} b_3 e^{-\frac{\lambda y'}{Re}} \right)}{a_1 + a_2 x + a_3 y + a_4 x y + \left( b_1 \cos \mu x' + c_1 \sin \mu x' \right) \left( b_2 \cos \sqrt{\lambda^2 - \mu^2} y' + c_2 \sin \sqrt{\lambda^2 - \mu^2} y' \right) \left( b_3 e^{-\frac{\lambda y'}{Re}} \right)} \right) \]  \hspace{1cm} (31)

Now applying inverse OST we get the exact solutions of 2D NSEs as

\[ u(x, y, t) = F(t) - F(0) + u(x', y', t') \]  \hspace{1cm} (32)
\[ v(x, y, t) = F(t) - F(0) + v(x', y', t') \]  \hspace{1cm} (33)

where \( x', y', t', u', v' \) will be determined from equations (9)-(13).

4. Conclusions

In this study, we have shown how to generate exact solutions of 2D NSEs by using OST and CHT with the help of separation of variables method. By using same method one can easily find out exact solutions 3D and 1D NSE. The method is simple and can be used to find exact solutions of the governing equations when we set initial and boundary conditions.
References


