Article

Decision Making in Agriculture: A Linear Programming Approach

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Abstract: Linear programming (LP) technique is relevant in optimization of resource allocation and achieving efficiency in production planning particularly in achieving increased agriculture production of food crops (Rice, Maize, wheat, Pulses and other crops). In this paper a Linear programming technique is applied to determine the optimum land allocation of 5 food crops by using agriculture data, with respect to various factors viz. Daily wages of labour and machine charges for the period 2004-2011. The proposed LP model is solved by standard simplex algorithm. It is observed that the proposed LP model is appropriate for finding the optimal land allocation to the major food crops.

Keywords: Land and resource allocation, Optimization techniques, Simplex Algorithm, Decision making

Mathematics Subject Classification Code (2010): 68M15, 62F15

1. Introduction

The agricultural planning problems are important from both social and economic points of view. They involve complex interactions of nature and economics. Agriculture contributes to nearly 25% of
Gross Domestic Product and about 70% of Indian population is dependent on agriculture for their livelihood (Fazile & Ashraf [2]). Agricultural planning is important in recent times due to the increased demand of agricultural commodity because of population increase. Due to the increase in population, there is always a need of more production to meet the ever increasing demand. One way of achieving high productivity is to increase the area under cultivation. Third world countries like India and others are losing land due to population growth and industrialization. As a result, the production of crop per unit area must be increased by proper utilization of resources. Planning of crops is the most crucial factor of agriculture planning. Crops planning depends on several resources like the availability of land, water, labor, and capital (Sarker and Quaddus, [12]). It also requires consideration of methods of irrigation, soil characteristics, cropping pattern, cropping intensity, topography, socio-economic conditions, climate, and many other factors. Farmers use a wide range of production systems, which result in large variations in productivity among farms. Agricultural economics which deals with scientific planning for agricultural development has become an important area of specialization in agriculture. Optimal crop pattern and production of food crops with maximum profit is important information for agricultural planning using optimization methods. Crop yield, man power, production cost and physical soil type are required to build the method. With optimization techniques available; such as Linear Programming (LP), Dynamic Programming (DP) and Genetic Algorithm (GA), it is LP model that is more popular because of the proportionate characteristic of the allocation problems.

Operations research (OR) models began to be applied in agriculture in the early 1950s. It was Waugh, [16] who first proposed the use of linear programming to establish least-cost combinations of feeding stuffs and livestock rations. The linear program minimizes the cost of the blend, while some specified level of nutritional requirements represents the model’s constraints. The founder of linear programming, George B. Danzig, published his first related work in 1947, i.e., just four years before Waugh’s publication. Heady [9] proposed the use of linear programming for determining optimum crop rotations on a farm. In this case, the objective function represents the gross margin associated with the cropping pattern, while constraints relate to the availability of resources such as land, labor, machinery, and working capital. Even though linear programs were the first OR models in agriculture, many other OR approaches have been widely used in farming over the last sixty years. Zhang et al. [19] had given a survey of applications of operations research in the area of agriculture, which includes farming, forestry, stock-raising, fishery, etc.

Linear Programming (LP) is utilized by all sorts of firms in making decisions about establishment of new industries and in deciding upon different methods of production, distribution, marketing and policy decision making. Linear Programming (LP) is perhaps the most important and best-studied optimization problem. A lot of real world problems can be formulated as linear programming problems.
The simplex algorithm developed by Dantzig [4], starts with a primal feasible basis and uses pivot operations in order to preserve the feasibility of the basis and guarantee monotonicity of the objective value. For LP models with ≥ or = type constraints, the problem of obtaining initial basic feasible solution is difficult as these problems lack feasibility at origin. The usual approach to solve such problems is to use either two-phase or Big-M method each of which involves artificial variables and the introduction of artificial variable brings artificiality in otherwise straightforward simplex method. Another example is the combined application of General Information System and linear programming to strategic planning of agricultural uses was carried out by (Campbell et al. [3]. Keith [10] suggested that in the current economic climate, linear programming could well be worth reconsidering as a Maximizing technique in farm planning. This particularly applies when it is used in conjunction with integer programming, which allows many of LP's problems to be overcome. Felix and Judith [6] used an LP model for farm resource allocation. They compared between the results obtained from the use of the LP model and the traditional method of planning and observed that the results obtained by using the LP model are more superior to that of obtained by traditional method. Ion and Turek [11] suggested LP method to determine the optimal structure of crops, different methods which take into account the income and expenditure of crops per hectare were used for optimizing profit. They observed that, after applying the econometric model the profit rose to 143% and costs reduced to 81%. Weintraub and Romero [1] analyzed the use of operations research models to assess the past performance in the field of agricultural and forestry and to highlight current problems and future directions of research and applications. In the agriculture part, they concentrated on planning problems at the farm and regional-sector level, environmental implications, risk and uncertainty issues, multiple criteria, and the formulation of livestock rations and feeding stuffs. Tanko. et al. [14] studies in optimum resource allocation using LP approaches have largely been attempted in many countries. Wankhade and Lunge [15] carried out a case study and observed that the LP model is appropriate for finding the optimal land allocation to the major crops.

The simplex method which is used to solve linear programming was developed by George B. Dantzig in 1947 as a product of his research work during World War II when he was working in the pentagon with the mill. Most linear programming problems are solved with this method. He extended his research work to solving problems of planning or scheduling dynamically overtime, particularly planning dynamically under uncertainty. Concentrating on the development and application of specific operations research techniques to determine the optimal choice among several coeaces of action, including the evaluation of specific numerical values, we need to construct or formulate mathematical model Hiller et al. [8] and Dantzig [4]. Conclusively, the development of linear programming has been ranked among the most important scientific advances of the mid-20th century and its assessment is
generally accepted. Its impact since 1950 has been extraordinary. Today it is the standard tool that has saved thousands or millions of dollars of many production companies and agricultural sector.

2. Methodology

A linear programming problem with “n” decision variables and “m” constraints can be mathematically modeled as (Taha [13], Zeleny, [18], Winston, [17] and Higle & Wallence, [7]).

Maximize \( Z = c_1x_1 + c_2x_2 + \ldots + c_nx_n \)

Subject to (s.t.)

\[ a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n \leq b_1 \]
\[ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \leq b_2 \]
\[ \vdots \]
\[ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n \leq b_m \]

\( x_j \geq 0, \ j = 1,2, \ldots, n \)

This can be written as,

Max \( Z = C^T X \)

Subject to

\( AX \leq b, \)
\( X \geq 0 \)

From the model above, X represent the vector of variables (to be determined) while C and b are vectors of known matrix of coefficient. The expression to be maximized is called the objective function (\( C^T \) in this case). The equation \( AX \leq b \) is the constraint which specifies a convex polytope over which the objective function is to be optimized. The coefficients \( c_1, c_2, \ldots, c_n \) are the unit returns for the coming from each production process \( x_1, x_2, \ldots, x_n \).

3. Problem Formulation

To formulate the problem mathematically, the following notations are used

\( Z = \) The objective function to be maximize.
\( x_j = \) Input Variables
\( c_i \) = Cost coefficients of the objective function \( Z \)

\( b_i \) = Maximum limit of the constraints.

\( a_{ij} \) = Coefficients of the functional constraint equations.

In general the planning models usually take the form

\[
\text{Maximize } Z = \sum_{i=1}^{n} c_i x_j
\]

Subject to

\[
\sum_{j=1}^{n} A_{ij} x_j \leq b_i \quad i = 1,2,\ldots,m
\]

\[
x_j \geq 0 \quad j = 1,2,\ldots,n
\]

where \( A_{ij} = [a_{ij}]_{m \times n}, \quad x_j = [x_{ij}]_{m \times 1} \quad \text{and} \quad b_i = [b_{ij}]_{n \times 1} \)

\( c_j, x_{ij}, b_i \in \mathbb{R} \)

If we convert above G.L.P.P in Standard form by using Slack Variables as \( x_{n+i} \)

\[
\text{Maximize } Z = \sum_{i=1}^{n} c_i x_j
\]

Subject to

\[
\sum_{j=1}^{n} A_{ij} x_j + x_{n+i} = b_i \quad i = 1,2,\ldots,m
\]

\[
x_j \geq 0 \quad j = 1,2,\ldots,n
\]

4. Solution Procedure for Maximizing Problem

In order to maximizing problem the following procedure is necessary.

1) Set up the inequalities describing the problem.
2) Convert the inequalities to equalities by adding slack variables.
3) Enter the equalities in a table for initial basic feasible solutions with all slack variables as basic variables.
4) Calculate $Z_j - C_j$ values for this solution where $C_j$ is objective function coefficients for variable $j$ and $Z_j$ represents the decrease in the value of the objective function that will result if one unit of the variable corresponds to the column of a matrix is brought into the basis.
5) Determine the entering variable by choosing the one with the highest negative value.
6) Determine the row to be replaced by dividing the quality column $b_i$ by their corresponding optimum column values and choosing the smallest positive quotient.
7) Compute the evolutes for the entering rows.
8) Compute values for the remaining rows.
9) Calculate $Z_j - C_j$ for this solution.
10) If there is positive $Z_j - C_j$ value, then optimal solution has been obtained otherwise go to next step’s optimal solution is obtained when all the entries in $Z_j - C_j A = \pi r^2$ positive or zero.

5. Numerical Illustration

To determine the optimum land allocation of 5 food crops by using agriculture data, with respect to various factors viz. Daily wages of labour and machine charges for the period 2004-2011. The data was collected from the Department of Economics and Statistics Statistical Digest [5], Government of Jammu & Kashmir (GoJK), Srinagar. The estimation of the coefficients $a_{11}, a_{12}, \ldots a_{mn}$ in (3.2) which are usually termed “production” coefficients, is probably the most difficult task in the formulation of the relevant mathematical model. Additional efforts in this respect are generally more important than in additional refinements in the mathematical approach. This additional effort is necessary, since if the input coefficients are too low, the subsequent plan will be non-feasible because it will require more resources than the available. If however, the input coefficients are too high, the farm- firm will find that surplus resources exist, and a better plan can be found. To obtain the production coefficients it is necessary to determine the amount of a particular input required to produce an acre of rice, wheat and soon. A typical matrix of these coefficients including the expected output per acre and the requirements is shown in table 1.
In our case, objective function is the output of various agriculture productions of food crops, inequalities is the Land / Capital/Labour (A) and Labour (B) and requirement is total. Now, our objective is to find the optimum land of food crops.

Table 1 represents in simplified manner the basic information necessary in order to construct a linear programming model of land utilization. This model, which in the interests of simplicity ignores livestock, is as follows:

\[
\begin{align*}
\text{Maximize } & \quad Z = 102.85X_1 + 114.84X_2 + 263.50X_3 + 34.13X_4 + 98.26X_5 \\
\text{Subject to } & \quad 773.40X_1 + 941.84X_2 + 823.13X_3 + 124.99X_4 + 89.20X_5 \leq 2752.56 \\
& \quad 22.03X_1 + 45.74X_2 + 63.47X_3 + 6.67X_4 + 10.91X_5 \leq 2409 \\
& \quad 10.08X_1 + 14.10X_2 + 17.33X_3 + 0.80X_4 + 7.32X_5 \leq 1069.7 \\
& \quad 13.09X_1 + 12.08X_2 + 18.08X_3 + 2.16X_4 + 9.07X_5 \leq 111 \\
& \quad x_1, x_2, x_3, x_4, x_5 \geq 0
\end{align*}
\]

Applying the simplex procedure for obtaining the optimum land of Food Crops through LINGO computer based software

Global optimal solution found.

Objective Value : 1375.996
Infeasibilities : 0.000000
Total solver iterations : 4
Model Class : LP

Total variables : 5
Nonlinear variables : 0
Integer variables : 0

Total constraints : 5
Nonlinear constraints : 0

Total nonzero : 25
Nonlinear nonzero : 0

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>VALUE</th>
<th>REDUCED COST</th>
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<tr>
<td>X1</td>
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<td>106.5676</td>
</tr>
<tr>
<td>X2</td>
<td>0.000000</td>
<td>102.2864</td>
</tr>
<tr>
<td>X3</td>
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<td>0.000000</td>
</tr>
<tr>
<td>X4</td>
<td>0.000000</td>
<td>0.1500135</td>
</tr>
<tr>
<td>X5</td>
<td>7.124985</td>
<td>0.000000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ROW</th>
<th>SLACK OR SURPLUS</th>
<th>DUAL PRICE</th>
</tr>
</thead>
<tbody>
<tr>
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<td>1375.996</td>
<td>1.000000</td>
</tr>
<tr>
<td>2</td>
<td>0.000000</td>
<td>0.1050571</td>
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<td>3</td>
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<tr>
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</tr>
<tr>
<td>5</td>
<td>0.000000</td>
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</tr>
</tbody>
</table>

The solution to this model yields the following information:

\( X_3 = 2.57 \) acres of wheat and \( X_5 = 7.11 \) acres of other food crops. The ultimate aim is to produce realistic agriculture planning model for the regions in order to examine in detail the effect of variations in prices and quantities.
6. Conclusions

It has been observed that for some LP problems simplex algorithm takes less number of iterations as compared to other algorithms. In the present study we proposed LP model for optimum land allocation to the 5 major food crops in agriculture. The solutions are obtained by Simplex algorithm. The total land used is found to be 2752.56 acres which is greater than 2409 acres than the land available for cultivation in the first season. The maximum profit achieved is Rs. 1376.00

References


