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Article

Pattern Association Using New Maximally Entangled States in a Two-qubit System

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Abstract: New set of maximally entangled states (Singh-Rajput MES), constituting orthonormal eigen bases, has been revisited and its superiority and suitability in the processes of quantum associative memory (QuAM) have been demonstrated. Using these MES as memory states in the evolutionary process of pattern storage, the suitability and superiority of these MES over Bell's MES have been demonstrated and it has been shown that, under the operations of all the possible memorization operators for a two-qubit system, the first two states of Singh-Rajput MES are useful for storing the pattern $|11>$ and the last two of these MES are useful in storing the pattern $|10>$ while Bell's MES are not much suitable as memory states in a valid memorization process. Recall operations of quantum associate memory (QuAM) have been conducted through evolutionary process in terms of unitary operators by separately choosing Singh-Rajput MES and Bell's MES as memory states for various queries and it has been shown that in each case the choices of Singh-Rajput MES as valid memory states are much more suitable than those of Bell's MES.

Keywords: Quantum Associate Memory (QuAM); Recall Operation; Bell's MES; Singh-Rajput MES; Memory States; Queries.

1. Introduction

The physically allowed degree of quantum entanglement [1] and mixture is a timely issue given that the entangled mixed states could be advantageous for certain quantum information situation [2]. The simplest non-trivial multi-particle system that can be investigated theoretically, as well as experimentally, consists of two qubits which display many of the paradoxical features of quantum mechanics such as superposition and entanglement. Basis of entanglement is the correlation that can exist between qubits. From physical point of view, entanglement is still little understood. What makes it too powerful is the fact that since quantum states exist as superposition, these correlations exist in superposition as well and when superposition is destroyed, the proper correlation is somehow communicated between the qubits. It is this communication that is the crux of entanglement. Entanglement is one of the key resources required for quantum computation [3-5] and hence the experimental creation and measurement of entangled states is of crucial importance for various physical implementations of quantum computers [6, 7]. By quantum entanglement we mean quantum correlation among the distinct subsystems of the entire composite system. For such correlated quantum systems, it is not possible to specify the quantum state of any subsystem independently of the remaining subsystems. The generation of quantum entanglement among spatially separated particles requires non-local interactions through which the quantum correlations are dynamically created [8] but our present knowledge of quantum entanglement is not at all satisfactory[9]. However, the functional dependence of the entanglement measures like concurrence [10, 11], i-concurrence[12] and 3-tangle[13] on spin-correlation functions have been established [14]. Protection of quantum states of open system from decoherence is essential for robust quantum information processing and quantum control in quantum computers. Recent papers concerning entanglement in quantum- spin systems address the questions about the maximum entanglement of nearest neighbour qubits belonging to a ring of N qubits in a translational invariant quantum state[15], the dependence of entanglement between two qubits on temperature and external magnetic field[16-18], and three-qubits XYZ- model[19,20] and XY-model[21].

Quantum Associative Memory (QuAM) is an important tool for pattern recognition, intelligent control and artificial intelligence. Ventura and Martinez have built [22] QuAM where the stored patterns are considered as the basis states of the memory quantum states. They used a modified version of well-known Grover's quantum search algorithm [23] in an unsorted database as the retrieval algorithm. To overcome the limits of this model to only solve the completion problem by doing data retrieving from the noisy data, Ezhov et al have used [24,25] an exclusive method of quantum superposition and Grover's algorithm with distributed queries. The QuAM is thus the realization of extreme condition of using many Hopfield networks [26], each storing a single pattern in parallel quantum universes. New situations like Quantum Hopfield Networks and Quantum Associative memory opened the doors for the development of Quantum Neural Networks (QNN) which are Artificial Neural Networks (ANN) functioning according to quantum laws. QuAM is the most promising approach to completion and it incorporates pattern association which consists of two very important processes: Pattern Storage (or Memorization) and Pattern Recall. Primary purpose of QuAM is to memorize a set of patterns for completion and it has the ability to generalize over patterns not seen during learning. General form of QuAM suggests that the database includes all basis states but some of them are not used and correspond to spurious memories. In our very recent paper [27] a new set of maximally entangled states (Singh-Rajput MES) constituting a very powerful and reliable eigen basis (Singh-Rajput Basis) (different from Bell's magic bases) of two-qubit systems has been constructed, the functional dependence of the entanglement measures on spin correlation functions has been established in terms of these MES, and the suitability and superiority of these MES, over Bell's MES [10], have been demonstrated [28,29] for pattern classification in a two –qubit system.

In the present paper this set of new maximally entangled states, constituting orthonormal eigen bases, has been revisited and its superiority and suitability in the processes of memorization and recalling of quantum associative memory (QuAM) has been demonstrated. Carrying out the storage element of QuAM by applying all possible memorizing operators for a two qubit system on Singh-Rajput MES one by one, the corresponding sets of modified memorized states have been obtained and it has been demonstrated that in all cases the coefficient of pattern $|11>$ is enhanced in the modified memorized states corresponding to first two states of Singh-Rajput MES and that of pattern $|10>$ is enhanced in the memorized states corresponding to last two of these MES. Thus it has been shown that under the operations of all the possible memorization operators for a two-qubit system the first two states of Singh-Rajput MES are useful for storing the pattern $|11>$ and the last two of these MES are useful in storing the pattern $|10>$ while Bell's MES are not suitable as memory states in a valid memorization process. The recall operations of quantum associate memory (QuAM), for a two-qunit system, have been conductedthrough evolutionary processes in terms of unitary operators by separately choosing Singh-Rajput MES and Bell's MES as memory states for various queries and it has been shown that in each case the choices of Singh-Rajput MES as valid memory states are much more suitable than those of Bell's MES. Evolutionary process of recall operation has been carried out in terms of modified Grover's search algorithm [35,36] using Fourier Discrete Transform (DFT) and the swapping operator and it has been shown that the maximally entangled nature and the orthonormal property of Singh-Rajput MES are fully retained under the operation of swapping operator emphasizing that Singh-Rajput MES provide a suitable choice as memory states for the memory recall mechanism of QuAM in evolutionary process. It has also been shown that the Bell's states loose their MES nature on being operated upon by the swapping operator and do not provide a suitable choice as memory states for memory recall mechanism of QuAM in evolutionary process. Examining the suitability of a MES as memory state in the evolutionary recalling processes for various different queries (I.e. partial pattern), it has been demonstrated that all the four states of Singh-Rajput MES are suitable as the valid memory states in the recall procedure with the queries '1?' and '0?' , where symbol ? represents 0 or 1, while only second state of Bell's MES may be suitable only with query '0?'. It has also been shown that the first and the fourth states of Singh-Rajput MES are most suitable choices of memory states for the queries '11' and '00' respectively, in the evolutionary recall procedure.

2. Necessary and Sufficient Conditions for a Two-qubit State to Be Maximally Entangled States (MES)

A general two-qubit state may be written as:

$$|\Psi> = \frac{1}{\sqrt{\gamma}}[\, a|00> \; +b|01> +c|10> \; +d|11>] \tag{2.1}$$

$$= \frac{1}{\sqrt{\gamma}}\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

where $\gamma = |a|^2 + |b|^2 + |c|^2 + |d|^2$ (2.2)

This state may also be written as:

$$|\Psi> = \frac{1}{\sqrt{(2\gamma)}}[\, i(a-d)|\emptyset_1> +(a+d)\; |\emptyset_2> +i(b+c)|\emptyset_3> \; +(b-c)\; |\emptyset_4>] \tag{2.3}$$

where

$$|\phi_1> = -\frac{i}{\sqrt{2}}(\,|00> -|11>); \quad |\phi_2> = \frac{1}{\sqrt{2}}(\;|00> +|11>)$$

$$|\phi_3> = -\frac{i}{\sqrt{2}}(\,|01> +|10>)\,; \quad |\emptyset_4> = \frac{1}{\sqrt{2}}(\;|01> -|10>), \tag{2.4}$$

are maximally entangled Bell's states [18] which satisfy the following conditions;

$$\sum_{\mu=1}^{4} |\phi_\mu><\phi_\mu| = 1 \text{ and } <\phi_\mu|\phi_\upsilon> = \delta_{\mu\nu} \tag{2.5}$$

showing that these states constitute the orthonormal complete set and hence form the eigen-basis (magic basis) [17] of the space of two level Q-bits. Any two-qubit state may be written in magic basis as:

$$|\psi> = \sum_{k=1}^{4} b_k\,|\phi_k>$$

with its concurrence defined as[17,18]

$$|\,C(|\psi>) = |\sum_{k=1}^{4} b_k^2\,| \tag{2.6}$$

If the concurrence $C(|\psi>) = 1$, the state is maximally entangled while for $C(|\psi>) = 0$, the state $|\psi>$ is not entangled at all and for

$$0 < C(|\psi>) < 1, \quad (2.7)$$

the state $|\psi>$ is partially entangled. Thus the concurrence of the state, given by equation (2.1) may be written as

$$C(|\Psi>) = \frac{2}{\gamma}|ad - bc| \quad (2.8)$$

Thus, for non-entangled state (i. e. separable state), we have:

$$ad = bc \quad (2.9)$$

and for partially entangled states,

$$0 < \frac{2|ad-bc|}{\gamma} < 1 \quad (2.10)$$

For the state $|\Psi>$, given by eqn. (2.1) to be maximally entangled state (MES), we have:

$$2|ad - bc| = |a|^2 + |b|^2 + |c|^2 + |d|^2 \quad (2.11)$$

or $|(a \mp d^*)|^2 + |(b \pm c^*)|^2 = 0$ (2.12)

This can be true either for;

$$d = a^* \ and \ c = -b^* \quad (2.13)$$

or for; $d = -a^* \ and \ c = b^*$ (2.14)

These are the necessary conditions for the state $|\Psi>$ of equation (2.1) to be maximally entangled. Thus, we get the following two sets of MES:

$$|\Psi_1> = \frac{1}{\sqrt{(2[|a|^2+|b|^2\}}}[a|00> +b|01> -b^*|10> +a^*|11>] \quad (2.15)$$

and $|\Psi_2> = \frac{1}{\sqrt{(2[|a|^2+|b|^2\}}}[a|00> +b|01> +b^*|10> -a^*|11>]$ (2.16)

Bell states (i.e. magic bases) given by equations (2.4) may readily be obtained from the state $|\Psi_1>$ of equation (2.15) on substituting:

$$(a = 1, b = 0); (a = -i, b = 0); (a = 0, b = 1); and\ (a = 0, b = -i) \quad (2.17)$$

For these sets of values of $a\ and\ b$, the state $|\Psi_2>$ of eqn. (2.16) gives $|\emptyset_1>\ and\ |\emptyset_4>$ with phase rotated by $\frac{\pi}{2}$ and $|\emptyset_2>\ and\ |\emptyset_3>$ with phase rotated by $-\frac{\pi}{2}$.

Other maximally entangled two-qubit states which form the orthonormal complete set (i.e. eigen basis) may be obtained as follows by putting $a = \pm 1\ and\ b = 1$ in state $|\Psi_2>$ of equation (2.16) and $a = 1, b = \pm 1$ in state $|\Psi_1>$ of equation (2.15);

$$|\psi_1> = \frac{1}{2}[-|00> +|01> +|10> +|11>], \quad (2.18)$$

$$|\psi_2> = \frac{1}{2}[|00> -|01> +|10> +|11>], \quad (2.19)$$

$$|\psi_3> = \frac{1}{2}[|00> +|01> -|10> +|11>], \quad (2.20)$$

$$|\psi_4> = \frac{1}{2}[|00> +|01> +|10> -|11>] \quad (2.21)$$

The concurrence for each of these states is unity and these states constitute the orthonormal set since

$< \psi_\mu | \psi_\nu > = \delta_{\mu\nu}$

and $\sum_{\mu=1}^{4} |\psi_\mu >< \psi_\mu| = I$ (2.22)

Thus the set of Bell states is not the only eigen basis (magic eigen basis) of the space of two-qubit system but the set of MES given by eqns. (2.18-2.21) also constitute a very powerful and reliable eigen basis of two-qubit systems. This is the new eigen basis and to differentiate it from the already known Bell's basis we have designated it in our recent paper [27] as **Singh-Rajput basis** for its possible use in future in the literature. The MES constructed in the form given by eqns. (2.18-2.21) have been correspondingly labelled as **Singh-Rajput states.** In this basis, various qubits of two-qubit states may be written as:

$$|00> = \frac{1}{2}\,[\,|\psi_2> + |\psi_3> + |\psi_4> - |\psi_1>],$$

$$|01> = \frac{1}{2}\,[\,|\psi_1> + |\psi_3> + |\psi_4> - |\psi_2>],$$

$$|10> = \frac{1}{2}\,[\psi_1> + |\psi_2> + |\psi_4> - |\psi_3>], \qquad (2.23)$$

$$|11> = \frac{1}{2}\,[\,|\psi_1> + |\psi_2> + |\psi_3> - |\psi_4>]$$

Substituting these relations in equations (2.4), Bell states may be constructed as follows in this new basis;

$$|\phi_1> = \frac{-i}{\sqrt{2}}[\,|\psi_4> - |\psi_1>];\ |\phi_2> = \frac{1}{\sqrt{2}}[\,|\psi_2> + |\psi_3>];$$

$$|\phi_3> = \frac{-i}{\sqrt{2}}[|\psi_4> + |\psi_1>];\ \ |\phi_2> = \frac{1}{\sqrt{2}}[\,|\psi_3> - |\psi_2>] \qquad (2.24)$$

Concurrence of each of Bell states in this basis also is unity showing the invariance of concurrence in different bases.

Condition (2.10) for partial entanglement shows that if any coefficient of qubits in the state $|\Psi>$, given by equation (2.1), is vanishing, then the state is necessarily partially entangled and its concurrence is $\frac{2}{3}$ if the sum of squares of moduli of non-zero coefficients is 3. For instance, let $b = 0, and\ |a|^2 + |c|^2 + |d|^2 = 3$, then the concurrence given by equation (2.8) becomes $\frac{2}{3}$ when $a = \pm 1, c = \pm 1\ and\ d = \pm 1$. It may be readily shown that all the states $\frac{1}{\sqrt{3}}[\pm|00> \pm|01> \pm|11>]$ are partially entangled with concurrence$= \frac{2}{3}$.

3. Pattern Storage in QuAM

The storage and recall mechanism of QuAM are fundamentally different from the traditional associative schemes such as Hopfield [26], bidirectional associative memory (BAM) [32], RAAM [33] etc. For the storage mechanism an input state consisting with equal amplitude is desired. The algorithm for constructing a coherent state over n qubits to represent a set of m patterns is implemented using a polynomial number (in length and number of patterns) of elementary operations over qubits. The key operator in this process is [22]

$$\hat{S}^P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \sqrt{(\frac{p-1}{p})} & -\frac{1}{\sqrt{p}} \\ 0 & 0 & \frac{1}{\sqrt{p}} & \sqrt{(\frac{p-1}{p})} \end{bmatrix} \tag{3.1}$$

where $m \geq p \geq 1$. It is obviously a unitary operator and hence the storage segment of QuAm through this operator is an evolutionary process. This evolutionary nature of storing process is necessary for the system to maintain coherent superposition that represents the stored patterns.

There is a different $\hat{\hat{S}}^P$ operator for each pattern to be stored. If we have n binary patterns each of length n, the quantum algorithm for storing the patterns requires a set of (2n+1) qubits, the first n of which stores the patterns and remaining (n+1) are used for book keeping which are restored to the string state after every storing iteration. For n=4 in a two qubit system the nine qubits are required first four of which constitute four maximally entangled states (Singh-Rajput MES) and the remaining five are their transformations after each iteration. In this case we have 4$\geq p \geq 1$. For different values of p the operator of eqn. (3.1) may be written as

$$\hat{S}^1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}; \quad \hat{S}^2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$\hat{S}^3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{\sqrt{2}}{\sqrt{3}} & -\frac{1}{\sqrt{3}} \\ 0 & 0 & \frac{1}{\sqrt{3}} & \frac{\sqrt{2}}{\sqrt{3}} \end{bmatrix}; \quad \hat{S}^4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ 0 & 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \tag{3.2}$$

Each iteration makes use of different $\hat{S}^P$ and results in another pattern being incorporated into the quantum system. Applying the memorization operator $\hat{S}^1$ of eqns. (3.2) on Singh-Rajput MES, given by eqns. (2.18)-(2.21), we have;

$$\hat{S}^1|\psi_1> = \frac{1}{2}[-|00> +|01> -|10> +|11>] = |\psi_1{}^I>, \tag{3.3a}$$

$$\hat{S}^1|\psi_2> = \frac{1}{2}[|00> -|01> -|10> +|11>] = |\psi_2{}^I>, \tag{3.3b}$$

$\hat{S}^1|\psi_3> = \frac{1}{2}[\,|00> + |01> - |10> - |11>] = |\psi_3{}^I>$, (3.3c)

$\hat{S}^1|\psi_4> = \frac{1}{2}[\,|00> + |01> + |10> + |11>] = |\psi_4{}^I>$, (3.3d)

where the resulting states $|\psi_1{}^I>, |\psi_2{}^I>$, $|\psi_3{}^I>$ and $|\psi_4{}^I>$ are not entangled at all, in view of condition (2.7) and eqn. (2.8), and hence the maximally entangled nature of Singh-Rajput MES are completely lost after the operation of the operator $\hat{S}^1$ and the magnitude of any pattern is not modified (except the phase change) as per requirement of the memorization process of QuAM in any state. Thus the operator $\hat{S}^1$ is not suitable for storing (i.e. memorizing) Singh-Rajput MES as valid memory states. Applying the operator $\hat{S}^2$of eqns. (3.2) on Singh-Rajput MES, we get

$\hat{S}^2|\psi_1> = -0.5\,|00> + 0.5|01> + 0.707|11> = |\psi_1{}^{II}>$, (3.4a)

$\hat{S}^2|\psi_2> = 0.5\,|00> - 0.5|01> + 0.707|11> = |\psi_2{}^{II}>$, (3.4b)

$\hat{S}^2|\psi_3> = 0.5\,|00> + 0.5|01> - 0.707|10> = |\psi_3{}^{II}>$, (3.4c)

$\hat{S}^2|\psi_4> = 0.5\,|00> + 0.5|01> + 0.707|10> = |\psi_4{}^{II}>$, (3.4d)

where all the resulting states $|\psi_1{}^{II}>, |\psi_2{}^{II}>, |\psi_3{}^{II}>$, and $|\psi_4{}^{II}>$ are entangled (though not maximally entangled) in view of eqn. (2.7) and condition (2.8). It is also shown by eqns. (3.4) that the coefficient of pattern $|11>$ is increased and that of pattern $|10>$ vanishes by the operations of the memorization operator $\hat{S}^2$ on the first and second states, $|\psi_1>$ and $|\psi_2>$, of Singh-Rajput MES while its operation on the last two states$|\psi_3>$ and $|\psi_4>$, of these MES enhances the coefficient of pattern $|10>$ and makes the coefficient of the pattern$|11>$ vanishing. Thus the choice of states $|\psi_1>$ and $|\psi_2>$ of Singh-Rajput MES as memory states may be useful in recalling the pattern $|11>$ in QuAM while that of states $|\psi_3>$ and $|\psi_4>$ may be found useful in the recalling of pattern $|10>$. Furthermore the variation in the values of coefficients in the superposition, given by eqns. (3.4), makes the resulting memory states the distributed memory [29,30] having several advantages in QuAM over the uniform memory states with equal coefficients of all patterns.

Similarly on applying the memorization operators $\hat{S}^3$ and $\hat{S}^4$ of eqns. (3.2) on the states of Singh-Rajput MES, we have

$\hat{S}^3|\psi_1> = -0.5\,|00> + 0.5|01> + 0.119\,|10> + 0.697|11>$
$= |\psi_1{}^{III}>$ (3.5a)

$\hat{S}^3|\psi_2> = 0.5\,|00> - 0.5|01> + 0.119\,|10> + 0.697|11>$
$= |\psi_2{}^{III}>$, (3.5b)

$\hat{S}^3|\psi_3> = 0.5\,|00> + 0.5|01> - 0.697\,|10> + 0.119|11>$
$= |\psi_3{}^{III}>$, (3.5c)

$\hat{S}^3|\psi_4> = 0.5\,|00> + 0.5|01> + 0.697\,|10> - 0.119|11>$

$= |\psi_4{}^{III}>,$ (3.5d)

where all the resulting states $|\psi_1{}^{III}>, |\psi_2{}^{III}>, |\psi_3{}^{III}>$ and $|\psi_4{}^{III}>$ are maximally entangled, in view of eqn. (2.7) and condition (2.8), with enhanced coefficient of pattern $|11>$ in memorised states $|\psi_1{}^{III}> and\ |\psi_2{}^{III}>$ and the enhanced coefficient of pattern $|10>$ in the memorised states $|\psi_3{}^{III}>$ and $|\psi_4{}^{III}>$. Thus with the memorization operator $\hat{S}^3$ also the first two states of Singh-Rajput MES may be useful for storing the pattern $|11>$ while the last two states of these MES are useful for storing the pattern$|10>$.

Similar results are obtained on applying the memorizing operator $\hat{S}^4$ on different states of Singh-Rajput MES;

$$\hat{S}^4|\psi_1> = -0.5\,|00> + 0.5|01> + 0.183\,|10> + 0.683|11>$$

$= |\psi_1{}^{IV}>,$ (3.6a)

$$\hat{S}^4|\psi_2> = 0.5\,|00> - 0.5|01> + 0.183\,|10> + 0.683|11>$$

$= |\psi_2{}^{IV}>$, (3.6b)

$$\hat{S}^4|\psi_3> = 0.5\,|00> + 0.5|01> - 0.683|10> + 0.183|11>$$

$= |\psi_3{}^{IV}>$ (3.6c)

$$S^4|\psi_4> = 0.5\,|00> + 0.5|01> + 0.683|10> - 0.183|11>$$

$= |\psi_4{}^{IV}>$ (3.6d)

where the resulting memorized states $|\psi_1{}^{IV}>, |\psi_2{}^{IV}>, |\psi_3{}^{IV}>$ and $|\psi_4{}^{IV}>$ are all maximally entangled, in view of eqn. (2.7) and condition (2.8), with the enhanced value of coefficient of pattern $|11>$ in the first two memorized states and the enhanced value of coefficient in the last two memorized states. Thus with the memorization operator $\hat{S}^4$ also the first two states of Singh-Rajput MES are useful for storing the pattern $|11>$ and the the last two states of these MES are suitable for storing the pattern$|10>$.

Applying the memorization operators of eqns. (3.2) on the Bell's MES, given by eqns. (2.4), we have

$$\hat{S}^1|\phi_1> = |\phi_3> = |\phi_1{}^{I}>; \quad \hat{S}^1|\phi_2> = |\phi_4> = |\phi_2{}^{I}>$$

$$\hat{S}^1|\phi_3> = \frac{-i}{\sqrt{2}}[|01> + |11>] = |\phi_3{}^{I}>;$$

$\hat{S}^1|\phi_4> = \frac{1}{\sqrt{2}}[|01> - |11>] = |\phi_4{}^{I}>$ (3.7)

$$\hat{S}^2|\phi_1> = -0.707i|00> - 0.5i|10> + 0.5i|11> = |\phi_1{}^{II}>;$$

$$\hat{S}^2|\phi_2> = 0.707|00> - 0.5|10> + 0.5|11> = |\phi_2{}^{II}>;$$

$$\hat{S}^2|\phi_3> = -0.707i|01> \ -0.5i|10> \ -0.5i|11> = |\phi_3{}^{II}>;$$

$$\hat{S}^2|\phi_4> = 0.707|01> \ -0.5|10> \ -0.5|11> = |\phi_4{}^{II}> \quad (3.8)$$

$$\hat{S}^3|\phi_1> = -0.707i|00> \ -0.41i|10> \ +0.58i|11> = |\phi_1{}^{III}>;$$

$$\hat{S}^3|\phi_2> = 0.707|00> \ -0.4|10> \ +0.58|11> = |\phi_2{}^{III}>;$$

$$\hat{S}^3|\phi_3> = -0.707i|01> \ -0.577i|10> \ -0.41i|11> = |\phi_3{}^{III}>;$$

$$\hat{S}^3|\phi_4> = 0.707|01> -0.577|10> -0.41|11> = |\phi_4{}^{III}>) \quad (3.9)$$

$$\hat{S}^4|\phi_1> = -0.707i|00> -0.354i|10> +0.612i|11> = |\phi_1{}^{IV}>;$$

$$\hat{S}^4|\phi_2> = 0.707|00> -0.354|10> +0.612|11> = |\phi_2{}^{IV}>;$$

$$\hat{S}^4|\phi_3> = 0.707i|01> +0.612i|10> +0.354i\ |11> = |\phi_3{}^{IV}>;$$

$$\hat{S}^4|\phi_4> = 0.707|01> +0.612|10> +0.354\ |11> = |\phi_4{}^{IV}> \quad (3.10)$$

where the first two of eqns. (3.7) show that the memorization operator $\hat{S}^1$ transforms the first two states of Bell' MES into third and fourth states respectively without any modification in the coefficient of any of the patterns in the superposition but inducing the patterns which were not present in the original states. The last two of the eqns. (3.7) show that this memorization operator creates two new states $|\phi_3{}^{I}>;$ and $|\phi_4{}^{I}>;$ none of which is entangled at all and hence the operator $\hat{S}^1$ is not the suitable choice as memorization operator for Bell' MES as memory states. Other sets of eqns. (3.8-3.10) show that all the other memorization operators $\hat{S}^2, \hat{S}^3$ and$\hat{S}^4$ of eqns. (3.2) enhances the coefficients of the pattern $|00>$ when first two of Bell's MES are chosen as memory states while these operators enhance the coefficients of pattern $|01>$ in the last two of Bell's MES but in each of the modified states, $\phi_\mu^{II}, \phi_\mu^{III}$and ϕ_μ^{IV} for $\mu = 1,2,3,4,$ a new pattern, not present in the initial memory state as any of Bell's MES, is created as the spurious or fictitious memory and none of these modified memory states is maximally entangled. Thus the Bell's states are not suitable as memory states for memorization process (storage algorithm) of QuAM.

It follows from this analysis that under the operations of all the possible memorization operators for a two-qubit system the first two states of Singh-Rajput MES are useful for storing the pattern $|11>$ and the last two of these MES are useful in storing the pattern $|10>$ while Bell's MES are not suitable as memory states in a valid memorization process.

4. Recall Operation of QuAM through Evolutionary Processes

The memory recall uses a modified Grover's search algorithm. Let us realise Grover's search algorithm using Discrete Fourier transform with the matrix elements given by [34]

$$F_{ab} = e^{2\pi iab/N}$$

where $N = 4, 0 \leq (a,b) \leq 3$. Thus we have the matrix of DFT as

$$F = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{bmatrix} \tag{4.1}$$

with transpose conjugate of this DFT matrix given as

$$F^{\dagger} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -i & -1 & i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & -i \end{bmatrix} = TF \tag{4.2}$$

where the matrix T swapping the rows of the DFT F is given by

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \tag{4.3}$$

The matrices F and T are obviously unitary matrices and the process related with the operations of these matrices are evolutionary. When the matrix T operates on different memories, we have

$$T\left|00>\right. = \left|00>\right.;\ T\left|01>\right. = \left|11>\right.;\ T\left|10>\right. = \left|10>\right.; T\left|11>\right. = \left|01>\right. \tag{4.4}$$

which may also be written as

$$T\left|?0>\right. = \left|?0>\right.;\ \text{where } ?= 0, 1 \tag{4.5a}$$

and $T\left|\alpha 1>\right. = \left|\alpha' 1>\right.$, (4.5b)

where $\alpha = 0{,}1\ and\ \alpha' = 0\ if\ \alpha = 1\ and\ \alpha' = 1\ if \alpha = 0$

Applying the Swapping- operator of equation (4.3) on Singh-Rajput MES, given by eqns. (2.18-2.21), we have:

$$T|\psi_1> = |\psi_1>;\ T|\psi_2> = |\psi_4>;$$

$$T|\psi_3> = |\psi_3>;\ T|\psi_4> = |\psi_2> \tag{4.6}$$

showing that the first and third states, $|\psi_1>$ and $|\psi_3>$ respectively, of Singh-Rajput Basis are self-swapped states while the second state $|\psi_2>$ is relabelled as fourth state $|\psi_4>$ and vice-versa, under swapping operator. Thus the maximally entangled nature and the orthonormal property of Singh-Rajput MES are fully retained under the operation of swapping operator of eqn. (2.4). In other words Singh-Rajput MES provide a suitable choice as memory states for the memory recall mechanism of QuAM.

On the other hand, if the swapping operator of eqn. (4.3) is operated upon the Bell's MES given by eqns. (2.4), then we get following new states:

$$|\alpha_1> = T|\phi_1> = -\frac{i}{\sqrt{2}}(|00> -|01>);$$

$$|\alpha_2> = \mathrm{T}|\phi_2> = \frac{1}{\sqrt{2}}(|00> +|01>);$$

$$|\alpha_3> = T|\phi_3> = -\frac{i}{\sqrt{2}}(\ |10> +|11>;$$

$$|\alpha_4> = T|\phi_4> = -\frac{1}{\sqrt{2}}\,(\,|10> - |11> \quad (4.7)$$

which are no more Bell's states and no more MES. In other words Bell's states loose their MES nature on being operated upon by the swapping operator of eqn. (4.3). Thus Bell's states do not provide a suitable choice as memory states for memory recall mechanism of QuAM.

Let us examine the suitability of a MES as memory state $|\psi_0>$ for recalling the memory associated with a given partial pattern by the recall mechanism [34]:

$$|\psi> = F^{-1}I_{\bar{0}}\,FI_M F^{-1}I_{\bar{0}}\,FI_{\emptyset}|\psi_0> \quad (4.8)$$

where the operator $I_{\emptyset}$ inverts the phase of the state $|\emptyset>$, operator I_M inverts the phase of any state representing a valid memory (it minimizes the effects of spurious memories that develop during recall process), the operator F is represented by the matrix given by eqn. (2.2) for a two-qubit system and its inverse F^{-1} is given by

$$F^{-1} = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -i & -1 & i \\ 1 & -1 & 1 & -1 \\ 1 & i & -1 & -i \end{bmatrix} \quad (4.9)$$

In this recall process any patterns in the stored memory, that match the query, have their phases inverted. Let the query be '0?', where ? represents unknown that matches either 0 or 1. In other words let the desired outcome be to recall the memory pattern whose first qubit is 0 in a two-qubit system. Let the first of Singh-Rajput MES, given by eqn. (2.18), be the memory state $|\psi_0>$. Let us write the given query as an operator $I_{0?}$. Then we have

$$\mathrm{I}_{0?}|\psi_0> = \mathrm{I}_{0?}|\psi_1> = \mathrm{I}_{0?}\,(\frac{1}{2}[\,-|00> \;+|01> +|10> \;+|11>])$$

$$= \frac{1}{2}[\,|00> \;-|01> +|10> \;+|11>]$$

$$= \frac{1}{2}\begin{bmatrix} 1 \\ -1 \\ 1 \\ 1 \end{bmatrix} = |\psi_1{}'> \quad (4.10)$$

Let us now perform the inversion effected by the operator sequence $-F^{-1}I_{\bar{0}}\,F$ where operators F and F^{-1} are given by eqns. (4.1) and (4.9) respectively and $I_{\bar{0}}$ inverts the phases of the memories representing the query. Thus we get

$$-F^{-1}I_{\bar{0}}\,F|\psi_1{}'> = \frac{1}{2}\begin{bmatrix} -i \\ 1 \\ i \\ 1 \end{bmatrix} = |\psi_2{}'> \quad (4.11)$$

where no spurious memory pattern (which is not present in the given memory state) is developed. Continuing with the operator sequence of eqn. (4.8) the phases of all valid memory states, involved in $|\psi_2{}'>$, are inverted as

$$\mathrm{I_M}|\psi_2{}' >= \frac{1}{2}\begin{bmatrix} i \\ -1 \\ -i \\ -1 \end{bmatrix} = |\psi_3{}' > \tag{4.12}$$

which gives

$$-F^{-1}I_{\bar{0}}\, F|\psi_3{}' > = \frac{1}{2}\begin{bmatrix} -1 \\ 1 \\ -1 \\ -1 \end{bmatrix} = -|\psi_1 > \tag{4.13}$$

where $|\psi_1 >$ is given by eqn. (2.18). Combining all eqns. (4.10)- (4.13), we may write the recall mechanism of eqn. (4.8) as

$$F^{-1}I_{\bar{0}}\, FI_M F^{-1}I_{\bar{0}}\, FI_{0?}|\psi_1 > = -|\psi_1 > \tag{4.14}$$

which shows that the first of Singh-Rajput MES as memory state is simply rotated by π under the recall process with query '01'.

In the similar manner we have

$$F^{-1}I_{\bar{0}}\, FI_M F^{-1}I_{\bar{0}}\, FI_{0?}|\psi_2 > = -|\psi_1 >; \tag{4.15}$$

$$F^{-1}I_{\bar{0}}\, FI_M F^{-1}I_{\bar{0}}\, FI_{0?}|\psi_3 > = |\psi_4 >; \tag{4.16}$$

$$F^{-1}I_{\bar{0}}\, FI_M F^{-1}I_{\bar{0}}\, FI_{0?}|\psi_4 > = |\psi_3 > \tag{4.17}$$

where eqn. (4.15) shows that with the second MES, given by (2.19), as valid memory state in the recall procedure of eqn. (4.8) with the query '0?', the output is the first MES given by eqn.(2.18) rotated by π in the similar manner as for the first MES as memory state. In other words the recall mechanism does not make any distinction between first two states of Singh –Rajput MES for the query '0?'. It is the most convenient and expected result for this query and hence any of these two states can be the suitable choice for the memory state in recall mechanism with the given query. Relations (4.16) and (4.17) show that the third and fourth MES given by eqns. (2.20) and (2.21) respectively, as the choice of valid memory states, are interchanged under the recall mechanism of eqn. (4.8) with the given query. However these states consist of the common memory patterns (with only sign change of one pattern) and hence no spurious, corrupted or fictitious memory pattern is generated by the given query under the recall procedure when the states of Singh-Rajput MES are used as the memory states. Thus all these states are the suitable memory states for the recall procedure with the given query '0?'.

Recall mechanism of eqn. (4.8), when applied on first of the Bell's states, given by eqn. (2.4), as memory states, yields

$$F^{-1}I_{\bar{0}}\, FI_M F^{-1}I_{\bar{0}}\, FI_{0?}|\phi_1 > = \frac{1}{\sqrt{2}}[|01 > +|10 >] = i\,|\phi_3 > \tag{4.18}$$

where $|\phi_3 >$ is the third Bell state of eqns. (2.4), which contains memory pattern not present in the memory state $|\phi_1 >$ and hence the out come of the recall procedure in eqn.(4.18) yields the spurious memories. Thus the first Bell' state is not suitable as memory state in the recall procedure for the given query. Similarly, for third, fourth and second Bell's MES, we have

$$F^{-1}I_{\bar{0}}\, FI_M F^{-1}I_{\bar{0}}\, FI_{0?}|\phi_3> = \frac{1}{\sqrt{2}}[|00> +|11>] = |\phi_2>; \qquad (4.19)$$

$$F^{-1}I_{\bar{0}}\, FI_M F^{-1}I_{\bar{0}}\, FI_{0?}|\phi_4> = \frac{i}{\sqrt{2}}[|00> -|11>] = -|\phi_1>; \qquad (4.\ 20)$$

and

$$F^{-1}I_{\bar{0}}\, FI_M F^{-1}I_{\bar{0}}\, FI_{0?}|\phi_2> = \frac{i}{\sqrt{2}}[|00> -|11>] = -|\phi_1> \qquad (4.21)$$

where eqns. (4.19) and (4.20) generate spurious memories in recalling mechanism through the given query and eqn. (4.21) shows that this procedure of recall from second Bell's MES as the memory state yields the inverted first MES. Thus among the Bell's MES only second state may be the valid choice as memory state with the given query.

Let us now make the query '1?', represented by the operator $I_{1?}$, where the unknown symbol may be either 0 or 1. This operator inverts the sign of third and fourth elements of column matrix representing the memory state used in the procedure of recall through this query. Thus for the first state of Singh-Rajput MES as the choice for memory state we have

$$I_{1?}|\psi_0> = I_{1?}|\psi_1> = \frac{1}{2}\begin{bmatrix}-1\\1\\-1\\-1\end{bmatrix} = |\psi_1{}'> \qquad (4.22)$$

which gives

$$-F^{-1}I_{\bar{0}}\, F|\psi_1{}'> = \frac{1}{2}\begin{bmatrix}-i\\1\\i\\1\end{bmatrix} = |\psi_2{}'>,$$

$$I_M|\psi_2{}'> = \frac{1}{2}\begin{bmatrix}i\\-1\\-i\\-1\end{bmatrix} = |\psi_3{}'>$$

and $-F^{-1}I_{\bar{0}}\, F|\psi_3{}'> = \frac{1}{2}\begin{bmatrix}1\\-1\\1\\1\end{bmatrix} = |\psi_2>$

Combining all these equations we have the recall mechanism of eqn. (4.8) as

$$F^{-1}I_{\bar{0}}\, FI_M F^{-1}I_{\bar{0}}\, FI_{1?}|\psi_1> = |\psi_2> \qquad (4.23)$$

which shows the generation of second of Singh-Rajput MES in the recall procedure with the given query $I_{1?}$ when the memory state is the first state of Singh-Rajput MES. The memory patterns in both these states are similar with the change of signs in the first and second elements of the matrices representing these states. No spurious states or the corrupt states are generated in this recall procedure and hence the first of Singh-Rajput MES can be a suitable and valid memory state for the given query. Similarly, we have

$$F^{-1}I_{\bar{0}}\, FI_M F^{-1}I_{\bar{0}}\, FI_{1?}|\psi_2> = \ |\psi_4>; \tag{4.24}$$

$$F^{-1}I_{\bar{0}}\, FI_M F^{-1}I_{\bar{0}}\, FI_{1?}|\psi_3> = \ -|\psi_4>; \tag{4.25}$$

$$F^{-1}I_{\bar{0}}\, FI_M F^{-1}I_{\bar{0}}\, FI_{1?}|\psi_4> = \ -|\psi_3> \tag{4.26}$$

Eqns. (4.23-4.26) show that the recall mechanism with the given query '1?' in QuAM with Singh-Rajput MES as valid memory states generate second state $|\psi_2>$ for the first memory state $|\psi_1>$ and gives the state $|\psi_4>$ for the memory state $|\psi_2>$ while the memory states $|\psi_3>$ and $|\psi_4>$ are relabelled as inverted states (rotated by π) $|\psi_4>$ and $|\psi_3>$ respectively. No spurious or fictitious or corrupted state is generated in the recall process with any of Singh-Rajput MES as the choice for memory state In the QuAM model for a two- qubit system.

If we choose the Bell's MES as the memory state then in the recall process for the given query '1?' we have the following output;

$$F^{-1}I_{\bar{0}}\, FI_M F^{-1}I_{\bar{0}}\, FI_{1?}|\phi_1> = \frac{1}{\sqrt{2}}\begin{bmatrix}0\\1\\-1\\0\end{bmatrix} = \ |\phi_4>; \tag{4.27}$$

$$F^{-1}I_{\bar{0}}\, FI_M F^{-1}I_{\bar{0}}\, FI_{1?}|\phi_2> = \frac{-1}{\sqrt{2}}\begin{bmatrix}0\\1\\1\\0\end{bmatrix} = \ -i|\phi_3>; \tag{4.28}$$

$$F^{-1}I_{\bar{0}}\, FI_M F^{-1}I_{\bar{0}}\, FI_{1?}|\phi_3> = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\0\\1\end{bmatrix} = \ |\phi_2>; \tag{4.29}$$

$$F^{-1}I_{\bar{0}}\, FI_M F^{-1}I_{\bar{0}}\, FI_{1?}|\phi_4> = \frac{-i}{\sqrt{2}}\begin{bmatrix}1\\0\\0\\-1\end{bmatrix} = \ |\phi_1> \tag{4.30}$$

Each recall process with the given query, represented by these equations, generates the state with the pattern different from that of the corresponding memory state. In other words the memory pattern of each output is not contained in the corresponding memory state and hence all the generated states in the recall process with Bell's MES chosen as memory states are spurious and fictitious memory states. Thus none of the Bell's MES is suitable choice for the valid memory state in the recall process with the given query '1?' in QuAM model.

Let us now consider the recall procedure with the query as point '11' or the pattern $|11>$. Let us choose the states of Singh-Rajput MES one by one for this recall. For the first of these MES as the chosen memory we have

$$\mathrm{I}_{11}|\psi_0> = \mathrm{I}_{11}|\psi_1> = \frac{1}{2}\begin{bmatrix}-1\\1\\1\\-1\end{bmatrix} = |\psi_1'>$$

which gives

$$-F^{-1}I_{\bar{0}}\,F|\psi_1'> = \frac{1}{2}\begin{bmatrix}1\\-1\\-1\\1\end{bmatrix} = |\psi_2'>,$$

$$\mathrm{I_M}|\psi_2'> = \frac{1}{2}\begin{bmatrix}-1\\1\\1\\-1\end{bmatrix} = |\psi_3'>$$

and $-F^{-1}I_{\bar{0}}\,F|\psi_3'> = \frac{1}{2}\begin{bmatrix}1\\1\\-1\\1\end{bmatrix} = |\psi_3>$

Combining all these equations we have the recall mechanism of eqn. (4.8) as

$$\mathrm{F^{-1}I_{\bar{0}}\,FI_M F^{-1}I_{\bar{0}}\,FI_{11}}|\psi_1> = |\psi_3> \quad (4.31)$$

where the generated state in the recall process is the third of Singh-Rajput MES with the similar memory patterns as contained in the chosen memory state $|\psi_1>$. In other words the recall procedure with the given query projects the first of Singh-Rajput MES as the third state $|\psi_3>$ without affecting the chances of observations of the given query $|11>$ and without generating any spurious or fictitious pattern. Thus the first of the Singh-Rajput MES, $|\psi_1>$, is the suitable choice as a valid memory state in recalling process with the given query.

Applying the recall procedure with the given query choosing others of Singh-Rajput MES, given by eqns. (2.19)-(2.21), as memory states one by one, we have

$$\mathrm{F^{-1}I_{\bar{0}}\,FI_M F^{-1}I_{\bar{0}}\,FI_{11}}|\psi_2> = \frac{1}{2}\begin{bmatrix}1\\-1\\1\\-1\end{bmatrix} \quad (4.32)$$

$$\mathrm{F^{-1}I_{\bar{0}}\,FI_M F^{-1}I_{\bar{0}}\,FI_{11}}|\psi_3> = \frac{1}{2}\begin{bmatrix}1\\-1\\-1\\1\end{bmatrix} \quad (4.33)$$

$$\mathrm{F^{-1}I_{\bar{0}}\,FI_M F^{-1}I_{\bar{0}}\,FI_{11}}|\psi_4> = -\frac{1}{2}\begin{bmatrix}1\\1\\1\\1\end{bmatrix} \quad (4.34)$$

where all the generated states in recall process are non-entangled and do not constitute the complete orthonormal set. In other words all the memory states lose their maximally entangled character in the process of recall with the given query. Thus all these states are the corrupt and fictitious states and hence none of these three states$|\psi_2>$, $|\psi_3>$ and $|\psi_4>$ of Singh-Rajput MES can be chosen as the valid memory state in the recalling process with the given query in QuAM model.

To check the validity of Bell's MES as the choice of memory state with this given query or partial pattern $|11>$, let us apply the recall process on these states one by one. Then we get

$$F^{-1}I_{\bar{0}}\,FI_M F^{-1}I_{\bar{0}}\,FI_{11}|\phi_1> = \frac{1}{\sqrt{2}}[|00> - |11>] = i|\phi_1>$$

$$F^{-1}I_{\bar{0}}\,FI_M F^{-1}I_{\bar{0}}\,FI_{11}|\phi_2> = \frac{-i}{\sqrt{2}}[|00> + |11>] = -i|\phi_2>$$

$$F^{-1}I_{\bar{0}}\,FI_M F^{-1}I_{\bar{0}}\,FI_{11}|\phi_3> = \frac{-1}{\sqrt{2}}[|01> - |10>] = -|\phi_4>$$

$$F^{-1}I_{\bar{0}}\,FI_M F^{-1}I_{\bar{0}}\,FI_{11}|\phi_4> = \frac{i}{\sqrt{2}}[|01> + |10>] = -|\phi_3> \qquad (4.35)$$

where the recall procedure represented by last two equations generates the states without the required pattern inherent in the given query and hence these equations give the fictitious states for the given query. Thus states $|\phi_3>$ and $|\phi_4>$ of the Bell's MES cannot be suitable choice as the valid memory states in the recall process with the given query. On the other hand the recall procedure, represented by first two of these equations (4.35), rotates the memory states $|\phi_1>$ and $|\phi_2>$ respectively by $\frac{\pi}{2}$ and $-\frac{\pi}{2}$ without affecting the chances of measurement of the required pattern in the given query. Thus these two states of Bell's MES can be chosen as valid memory states in the recall process with the given query.

Now let us apply recall procedure for Singh-Rajput MES as memory states with the query of point '00' representing the partial pattern $|00>$. For the first of these MES we have

$$I_{00}|\psi_0> = I_{00}|\psi_1> = \frac{1}{2}\begin{bmatrix}1\\1\\1\\1\end{bmatrix} = |\psi_1{}'>$$

which gives

$$-F^{-1}I_{\bar{0}}\,F|\psi_1{}'> = \frac{1}{2}\begin{bmatrix}1\\1\\1\\1\end{bmatrix} = |\psi_2{}'>,$$

$$I_M|\psi_2{}'> = \frac{1}{2}\begin{bmatrix}-1\\-1\\-1\\-1\end{bmatrix} = |\psi_3{}'>$$

and$-F^{-1}I_{\bar{0}}\,F|\psi_3{}'> = -\frac{1}{2}\begin{bmatrix}1\\1\\1\\1\end{bmatrix}$

Combining all these equations, we get the recall mechanism of eqn, (4.8) as

$$F^{-1}I_{\bar{0}}\,FI_M F^{-1}I_{\bar{0}}\,FI_{00}|\psi_1> = -\frac{1}{2}[|00> + |01> + |10> + |11>] \qquad (4.36)$$

where the state generated in the recalling procedure is neither maximally entangled nor the element of an orthonormal set of states. Thus the generated state in the recall mechanism is the fictitious and corrupt state and hence the first state $|\psi_1>$ of Singh-Rajput MES cannot be a choice as valid memory state in the recall procedure with the given query.

Choosing the second and third states $|\psi_2>$ and $|\psi_3>$, given by eqns. (2.19) and (2.20) respectively, as the memory states in the recall mechanism with the given query, we have

$$F^{-1}I_{\bar{0}}\, FI_M F^{-1}I_{\bar{0}}\, FI_{00}|\psi_2>=-\frac{1}{2}[-|00>-|01>\ +|10>+|11>]$$

and

$$F^{-1}I_{\bar{0}}\, FI_M F^{-1}I_{\bar{0}}\, FI_{00}|\psi_3>=-\frac{1}{2}[-|00>-|01>\ +|10>+|11>] \quad (4.37)$$

where the same state generated in both recall equations is corrupt state which is neither maximally entangled nor the element of an orthonormal set of states. Thus none of these states $|\psi_2>$ and $|\psi_3>$ of Singh-Rajput MES can be a suitable choice as a valid memory state in the recall procedure with the given memory. On the other hand when we choose the fourth state $|\psi_4>$ of this set of MES as the memory state then we get

$$F^{-1}I_{\bar{0}}\, FI_M F^{-1}I_{\bar{0}}\, FI_{00}|\psi_4>=-\frac{1}{2}[-|00>+|01>\ +|10>+|11>]=|\psi_1> \quad (4.38)$$

where the recall procedure transforms the memory state $|\psi_4>$ in to the first state $|\psi_1>$ with the nature of maximal entanglement and orthonormal property left intact without affecting the chance of observation of the pattern represented in the given query. Thus the fourth state$|\psi_4>$ of Singh-Rajput MES is most suitable as a valid memory state in the recall procedure with the given query.

Choosing various states of Bell's MES, given by eqns. (2.4), as memory state one by one in the recall procedure of QuAM with the given query as partial pattern $|00>$ we have

$$F^{-1}I_{\bar{0}}\, FI_M F^{-1}I_{\bar{0}}\, FI_{00}|\phi_1>=\frac{1}{\sqrt{2}}[|00>\ +|11>]=|\phi_2>; \quad (4.39)$$

$$F^{-1}I_{\bar{0}}\, FI_M F^{-1}I_{\bar{0}}\, FI_{00}|\phi_2>=\frac{1}{\sqrt{2}}[|00>\ -|11>]=i|\phi_1>; \quad (4.40)$$

$$F^{-1}I_{\bar{0}}\, FI_M F^{-1}I_{\bar{0}}\, FI_{00}|\phi_3>=-\frac{i}{\sqrt{2}}[|01>\ -|10>]=|\phi_3>; \quad (4.41)$$

$$F^{-1}I_{\bar{0}}\, FI_M F^{-1}I_{\bar{0}}\, FI_{00}|\phi_4>=\frac{1}{\sqrt{2}}[-|01>\ +|10>]=-|\phi_4> \quad (4.42)$$

where eqns. (4.41) and (4.42) representing recall procedure with Bell's MES $|\phi_3>$ and $|\phi_4>$ yields the states without any possibility of observing the given query point and hence these states cannot be chosen as the valid memory states in recall procedure. Recall eqn. (4.39), with the first Bell's state as the memory state, yields the second state of Bell's MES without affecting the chance of observation of the given partial pattern. Thus this Bell's state $|\phi_1>$ may be a suitable choice of the memory state in the recall process with given query.

It follows from the foregoing analysis that all the four states of Singh-Rajput MES are suitable as the valid memory states in the recall procedure with the queries '1?' and '0?' , where symbol ? represents 0 or 1, while only second state of Bell's MES may be suitable only with query '0?'. It has also been shown here that the first and the fourth states of Singh-Rajput MES are most suitable choices of memory states for the queries '11' and '00' respectively, in the evolutionary recall process.

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