Systematics of Ground State Lifetime for Beta Decay

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Abstract: Lifetime measurements for ground-state β-decay are systematized by using a semi-empirical relation. This semi-empirical correlation has a resemblance to the radioactive decay equation. This approach has been successfully applied to β-decay cases covering ~ 10 orders of magnitude in half-life. Nearly 250 measured half-life values have been analyzed. The present systematics allows predicting lifetime for a few possible cases of β-decay, which have not been determined experimentally yet.

Keywords: Half-life, β-decay, \(N_pN_n\) parameterization

1. Introduction

It is well known that various properties of nuclei may be correlated with the number of valence nucleons in a simple manner. A prime example is the \(N_pN_n\) parameterization, whose importance was first demonstrated by Casten[1-3] in connection with the role of the neutron-proton interaction in the growth of deformation away from shell closures. Here, \(N_p\) and \(N_n\) indicate the number of valence particles/holes for protons and neutrons, respectively. It is considered that if the number of valence nucleons lie below the middle of a major shell, they are considered as particles; otherwise they are taken to be holes.

Many developments in this theme have been made subsequently. Simplified parameterization of various nuclear properties can be obtained if they are plotted as a function of \(N_pN_n\) or some variants of...
them [1-19]. One kind of variant ($\xi_n$) has been considered for $\beta$-decay half-life systematics in Ref. [20] as

$$\xi_n = \frac{N_n}{(N_n)_{\text{max}}} \quad (1)$$

For a chain of isotopes, the denominator of $\xi_n$, $(N_n)_{\text{max}}$, is calculated taking the maximum $N_n$ value for the chain, which obviously corresponds to the middle of a shell. This normalized value represents the fraction of the integrated p-n interaction with respect to its maximum value in the chain of isotopes for a given shell.

The systematics of $\beta$-decay lifetime of the ground state of various isotope chains for even-even and odd-even nuclei was presented in Ref [20]. The beta decay lifetimes expressed as a function of the number and nature of valence particles. When the logarithm of the half-life ($\log \tau$) is plotted against the parameter $\xi_n$, the isotopes of all the elements show similar trend (with some exceptions). For a given $\xi_n$, the $\log \tau$ values shift among the isotopic chains in such a way that they coincide with $\log \tau$ values corresponding to a particular $\xi_n$. This emphasizes the similarity in trend. Although the trend is not exactly identical for all the elemental chains, one can see that the values fall within a band. In a valence region, the variation of the lifetime with the above mentioned ratios follow similar curves, indicating the possible role of the ratio. The facts mentioned above have stimulated us to find some correlation among the ground state lifetimes of $\beta$-decay and $\xi_n$

2. Formalism and Results

2.1. Parameters $\delta_n$ and $\delta_p$

The ground state decay contains nuclear structure effects, and maybe thought to depend on the overlap between the actual ground state configuration of the parent and the daughter state(s). Therefore, besides other quantities, lifetime should depend on $N_n^p/N_p^p$ the number of neutron/proton (particles/holes) in the parent nucleus, and $N_n^d/N_p^d$ the number of neutron/proton (particles/holes) in the daughter nucleus. The changes in the $N_n/N_p$ value depend on two factors, one is the way a nucleus decays i.e. either (EC/$\beta^+$ or ($\beta^-$) and second is whether the neutrons/protons are particle-like or hole-like?

It has been observed that the difference $\delta_n = N_n^d - N_n^p$, and $\delta_p = N_p^d - N_p^p$ has a character, which is same for all the nuclei showing a similar trend for isotopic/isotonic chain. It can be shown that a value of +1 (-1) corresponds to increase (decrease) of lifetime with $\xi_n/\xi_p$, where $\xi_p = \frac{N_p}{(N_p)_{\text{max}}}$. The values of these quantities ($\delta_n$ and $\delta_p$) for various valence regions have been given in the last columns of Table-1. In our notation, P and H refer to particle and hole type nucleons, respectively. For a valence
class, the first and the second letters refer to the natures (P or H) of the valence proton and neutron, respectively. A suffix ‘d’ indicates that the valence protons and neutrons occur in different shells.

Table 1: Trends of lifetime in different valence regions, decay mode and the parameter \( \delta_n, \delta_p \).

<table>
<thead>
<tr>
<th>Valence Regions</th>
<th>Trend with ( \xi_n )</th>
<th>Trend with ( \xi_p )</th>
<th>Decay Mode</th>
<th>( \delta_n = N_n^d - N_n^{p} )</th>
<th>( \delta_p = N_p^d - N_p^{p} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>P-P</td>
<td>Increasing</td>
<td>Decreasing</td>
<td>EC/( \beta^+ )</td>
<td>+1</td>
<td>-1</td>
</tr>
<tr>
<td>H-H</td>
<td>Decreasing</td>
<td>Increasing</td>
<td>EC/( \beta^+ )</td>
<td>-1</td>
<td>+1</td>
</tr>
<tr>
<td>P-H</td>
<td>Increasing</td>
<td>Increasing</td>
<td>( \beta^- )</td>
<td>+1</td>
<td>+1</td>
</tr>
<tr>
<td>P-P(_d)</td>
<td>Decreasing</td>
<td>Increasing</td>
<td>( \beta^- )</td>
<td>-1</td>
<td>+1</td>
</tr>
<tr>
<td>H-P(_d)</td>
<td>Increasing</td>
<td>Increasing</td>
<td>EC/( \beta^+ )</td>
<td>+1</td>
<td>-1</td>
</tr>
</tbody>
</table>

2.2. Proposed Semi-empirical Correlation

The proposed semi-empirical correlation on the basis of the smooth variation of \( \log \tau \) values against the parameter \( \xi_n/\xi_p \) within valence regions is given by

\[
\log \tau_{N_l} = \log \tau_{N_h} - \frac{\delta_{n/p} f_s}{(N_{n/p})_{\text{max}}} (N_h - N_l),
\]

(2)

Where \( \tau_{N_h} \) is the lifetime of isotope/isotone having highernumber of \((N_n/N_p)\), \( \tau_{N_l} \) is the lifetime of isotope/isotone having lowernumber of \((N_n/N_p)\) and \( f_s \) is structure dependent parameter. The \( f_s \) is mostlyidentical for an isotopic/isotonic chain for a valence region, if the nuclear structure follow identical behavior. The \( \delta_{n/p} \) is either \( \delta_n \) or \( \delta_p \) and values are given in Table-1. The \((N_{n/p})_{\text{max}}\) is either \((N_n)_{\text{max}}\) or \((N_p)_{\text{max}}\).

In the present investigation, it has been also observed that the proposed correlation is successfully applicable to a number of isotopic/isotonic sequences of \( \beta^- \)-decay nuclides, and, therefore, it has been used to systematize the \( \beta^- \)-decay half-lives of all known cases of ground-state. To keep the figure easily readable, we have not included all the elements in the region \( Z = 30-80 \), though we found that they follow a similar behavior. The data used in this investigation is taken from the ENSDF database [21].

To understand the trends, we have taken a few examples for the valence regions. For P-P region, we choose the even-even elements from \( Z = 30-34 \). All these elements have at least three isotopes which undergo \( \beta^- \)-decay in the P-P valence region. The \( \log \tau \) values for \( Z = 30-36 \) elements are plotted against the parameter \( \xi \) in such a way, the \( \log \tau \) values for \( ^{62}\text{Zn}(N_n=4) \) was taken as the experimental \( \log \tau \) value and the values for \( ^{58,60}\text{Zn} \ (N_n=0, 2) \) were calculated using the proposed correlation formula by assuming the value of \( f_s \) equal to 15. A comparison between the experimental \( \log \tau \) values and \( \log \tau \) values.
calculated using proposed correlation (equation 2) has been shown in Figure 1. The experimental log $\tau$ values were shown with filled symbol and co-related values with unfilled symbols.

![Figure 1](image-url)

**Figure 1:** (Color online) The logarithm values of half-life of ground-state $\beta$-decay with respect to $\xi_n$. The atomic number ($Z$) of isotopic chains is indicated in the figure. The experimental data are shown with filled symbols while open symbols indicate the data evaluated by correlation method. Structure factor $f_s$ is mentioned in order of 0.1 times. The lines are only for guiding the eye. See text for details.

For Ge isotopic chain, the log $\tau$ values for $^{68}\text{Ge}(N_n=8)$ was taken as experimental log $\tau$ value and the values for $^{62-66}\text{Ge}(N_n=2, 4, 6)$ were calculated by the correlated formula by assuming the $f_s$ value 15. For Se isotopic chain, the log $\tau$ value for $^{72}\text{Se}(N_n=10)$ was taken as experimental log $\tau$ and the values for $^{68,70}\text{Se}(N_n=6, 8)$ were calculated by the correlated formula by assuming the $f_s$ value 12. The correlated log $\tau$ values are in substantial agreement with the experimental values.

For H-P$_3$ valence region, following the same criterion as in P-P region, we observed that the log $\tau$ values follow a similar trend. To make the figure simple and clear, only the log $\tau$ values for $Z=74 - 78$ elements are shown. The log $\tau$ value for $^{176}\text{W}(N_n=22)$ was taken as experimental log $\tau$ value and the values for $^{172-174}\text{W}(N_n=16 - 20)$ were calculated by the correlated formula by assuming the $f_s$ value 7.
and $f_s$ value 5.5 for $^{162-170}$Os($N_n$=6-14). The log $\tau$ value for $^{180}$Os ($N_n$= 22) was taken as experimental log $\tau$ value and the values for $^{168-178}$Os($N_n$=10-20) were calculated by the correlated formula by assuming the $f_s$ value 5 and $f_s$ value 11.2 for $^{162-166}$Os($N_n$=4,6,8). Clearly, two values of $f_s$ are expected due to subshell effect. The log $\tau$ values for $^{182}$Pt($N_n$=20) was taken the experimental log $\tau$ value and the values for $^{176-180}$Pt($N_n$=6-14) were calculated by the correlated formula by assuming the $f_s$ value 5 and $f_s$ value 10 for $^{166-174}$Pt($N_n$=6-14). So, one can see that log $\tau$ values calculated using proposed correlation show reasonable match with experimental log $\tau$ values with few exceptions. Although the co-related values are not exactly identical for all the elemental chains, one can see small variation.

Finally, based on this present systematic study, the above correlation can be written in exponential form as

$$\tau_{N_l} = \tau_{N_h} \exp\left[-\frac{\delta_{n/p}f_s}{(N_{n/p})_{max}}(N_h - N_l)\right]$$

This equation has a resemblance to the radioactive decay equation $N = N_0 \exp(-\lambda t)$. It may consider that the role of $\frac{\delta_{n/p}f_s}{(N_{n/p})_{max}}$ is treated as decay constant for the valence region as $\lambda$ is decay constant in radioactive decay equation and $(N_h - N_l)$ is equivalent to time (t). The present correlation allows predicting lifetime for a few possible cases of $\beta$-decay radioactivity not yet experimentally determined.

3. Conclusion

The $\beta$-decay half-lives for ground states of a large number of isotopes/isotones series are seen to exhibit simple variations with the ratio of the numbers of valence neutrons/protons (particles or holes) and the maximum possible valence neutrons/protons in medium and heavy nuclei. The variations display distinct and unique trends in different valence regions. A semi-empirical correlation was used to calculate the lifetime of isotopes/isotones sequences within various valence regions. The calculated lifetimes are substantial agreement with the experimental values. This semi-empirical correlation has a resemblance to the radioactive decay equation. The proposed correlation in the present investigation has encouraged us to make predictions for a few cases of $\beta$-decay not yet determined experimentally.

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References
