The Polynomial Fitting Method for Correcting Total Field Archaeo-magnetic Data against Diurnal Effects

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Abstract: Some simplified adjustments have been made in the existing method of diurnal correction as applied to magnetic total field data acquired on land. The base station data required for the diurnal correction were obtained by repeated measurements of the magnetic total field at the fixed point throughout the course of the survey. Low order polynomials were fitted to the time-variant base station data using the least-squares technique, and some statistical analysis were applied to determine the best-fit model. The base station field was determined using the base model and the measurement times at the grid points (the field stations). The diurnal correction to be subtracted from each field measurement is the corresponding discrete value of the base field less the constant of its function. The method has been tested on magnetic total field data acquired from Ugbenlenabo in Lejja archaeological site, south-eastern Nigeria. The result obtained has been compared qualitatively with that of the conventional method of diurnal correction using the same set of data.

Keywords: Diurnal correction, archaeo-magnetic, polynomial fitting, statistical analysis.

1. Introduction

Measurement of the magnetic total field at a fixed location throughout the day reveals that there are temporal changes not related to any subsurface magnetic sources. Such changes are attributed
primarily to external sources such as the electric currents in the ionosphere and magnetosphere caused by the solar activities (Blakely, 1996; Campbell, 1997; Lowrie, 1997; Richmond, 2007). For this reason, the observed intensity of the external magnetic field (diurnal or daily variation) is said to be a function of the solar activity which is also latitude dependent. The magnetic measurements at the fixed location at night is relatively constant, and as revealed by the results obtained by Okeke and Hamano (2000), such amplitudes at the equatorial region begin to rise at about 6.00 to 7.00 am from the relatively constant values during the night hours, peaks during the day at about local noon and fall gradually towards evening. It can rise to as much as 30 nT or more at the earth’s surface on solar quiet days (Nettleton, 1976; Telford et al., 1990; Ogah, 2012). On the other hand, transient magnetic storm which is also caused by solar activities can result sudden changes in the external magnetic field to as much as 100 nT to 1000 nT during which it is advisable to discontinue magnetic surveys.

The diurnal variation has serious impact on magnetic interpretation because its magnitude often exceeds the accuracy demands of most magnetic surveys, particularly engineering and archaeological prospecting (Schmidt, 2002; Piro et al., 2007; Godio and Torino, 2005). Such effects must therefore be removed from the total field data. Three main approaches have been used (Dobrin, 1976; Dobrin and Savit, 1988; Telford et al., 1990; Nabighian et al., 2005). These include: (i) the application of gradiometer survey, (ii) the use of two magnetometers, one as a base meter and the other as a field meter, and (iii) use of a single magnetometer that must be returned to the base station for repeated measurements throughout the period of the survey. The first two options are for well funded projects. For some projects such as archaeological prospecting that seldom attract adequate funding (Clark, 1990), the third option may be the only option.

The magnetic total field data acquired by repeated measurements at the base station are used to correct observations made at the field stations. The conventional method of making correction for the diurnal variation is to obtain the difference between any two magnetic observations at the base station and subtract or add it to the observations made at the grid points (or field stations) within that time interval according to their time of observations. Such approach assumes a linear variation in the magnetic field recorded within the time interval. That is, fitting a linear function to the set of data that is better defined by a curvilinear function. Therefore the conventional method can only remove part of the effects of the diurnal variation. The major background noise will still remain superimposed on the total magnetic field data.

The aim of this work is to correct magnetic total field data for diurnal effect by fitting low order polynomial models to base station data using the least-squares approach. The fitting process uses varying powers of the approximating polynomials to calculate the values of the coefficients for each model. Further statistical calculations are needed to determine the best fit model out of a few candidate
models (Montgomery and Peck, 1992; Press et al., 1992; Aster et al., 2005). The time of measurements at the field stations and the computed parameters of the model are used to calculate the discrete amplitudes of the magnetic field at the base station that correspond with the time field observations were made at the grid points. The diurnal corrections to be subtracted from the field data are the fluctuations of the base field about the mean, the more stable values recorded before 7.00 am, which corresponds with the base station field less the constant component of the polynomial model.

2. Methodology

Assuming that the magnetic field at the base station can be represented by a polynomial of order m given by

$$B(t) = c_0 + c_1 t + c_2 t^2 + \ldots + c_m t^m$$

(1)

where $c_j$, $j = 0, 1, 2, \ldots, m$ are coefficients of the polynomial, while $t$ represents the measurement time. If repeated measurements are made $n$ different times at a fixed location, the measured field can be plotted against time of measurement as shown in the synthetic example of figure 1, where it is assumed (as earlier stated) that the diurnal variation is relatively stable at night (Nettleton, 1976; Lowrie, 1997; Okeke and Hamano, 2000; Ogah, 2012) but increases with the rising sun, peaks about midday and then decreases towards evening hours. The shaded portion of the figure describes the variation about the mean value (the horizontal line) of the magnetic field recorded at the fixed point. The curve has been extended beyond $t = 0$ in order to reflect the relatively stable field at the night hours. A common regression problem is finding parameters $c_j$’s in equation (1) that best fit a set of base station data with $n > m$. Using method of least-squares, the system of equation in such problem can be represented (Menke, 1989; Aster et al, 2005) as

$$G x = d$$

(2)

where $G$ is an $n \times m$ matrix of the powers of the independent variable $t$, $x$ is a column vector matrix of the $m$ model parameters, and $d$ is a data vector of $n$ observations. In other to obtain a computational

![Fig. 1: A prototype of the variation of magnetic field at a fixed location near the magnetic equator (Adapted from Ogah, 2012)](image-url)
formula for the calculation of the model parameters, the transpose of the matrix G can be computed and multiplied by G to obtain a square matrix given by \(G^T \cdot G\), which if substituted back into equation (2) gives the matrix products
\[
(G^T \cdot G) \cdot x = G^T \cdot d
\]
That is,
\[
x = (G^T \cdot G)^{-1}(G^T \cdot d).
\]
(3)

The column vector \(x\) contains the computed values of the unknown model parameters \(c_0, c_1, c_2, \ldots, c_m\). The technique of fitting polynomial curves to base station data obtains the values of \(x\) in equation (3), and substitutes them into equation (1) to produce the base station model equation.

3. Field Example

The magnetic total field data were acquired on a regular 1 x 1 metre grid covering a total area of 28 by 70 metres in Ugelenabo-Lejja archaeological site, south-eastern Nigeria. A proton precession magnetometer (Geometrics’ model G-856 AX) with resolution of 0.1 nT was used with magnetic sensor height fixed at 1 metre above the ground surface. A base station was established at mid point of a base line which was a straight line from which all the profiles were offset. The data acquisition which took place on the 4\textsuperscript{th} November, 2009, began with the first measurement at the base station at 8:11:30 am local time. This was followed by measurements along the profiles (oriented north-south), using bidirectional traversing method. The base station measurement was repeated after every two profiles (mean time of about 14 minutes) with each pair of base measurements corresponding with the beginning and end of a traverse.

Low order polynomial functions of varying degrees between 2 and 4 were fitted to the base station data using the least-squares technique. The graphs of the observed and the calculated base station fields are shown in figure 2 with the time axis made to begin from zero by subtracting the initial time (8.19167 hr) from all the time readings. In order to determine the model that best fitted the data, the test of significance of regression (F-test), was conducted in which the ratio of variances (\(F_o\)) of any two adjacent models was computed using the prediction error method of Menke (1989). If a model with one additional parameter higher than the preceding one produced \(F_o\) value much greater than unity, it was taken as a more acceptable one. This procedure was applied with caution, avoiding models of higher order (Montgomery and Peck, 1992) so as to avert over-fitting of the model to the data which could lead to formation of an ill-conditioned matrix (Press \textit{et al.}, 1992; Golub and Loan, 1996). After all, the so called best-fit parameters may not necessarily be the best predictor of the base station data. The results displayed in table 1 depict the cubic polynomial model that produced \(F_o\) value of 2.55 as the most appropriate model for this set of base station data.
Using the selected model, and the measurement time at each grid point, estimate of the base station observation corresponding with exact time of each observation at the grid station was computed. The diurnal correction to be subtracted from any magnetic total field data was therefore the difference between the corresponding base station field value and the constant component of the polynomial model, which is the intercept on the magnetic field axis of figure 2(b).

Table 1: Estimated model parameters and test of significance of added terms

<table>
<thead>
<tr>
<th>Order of polynomial</th>
<th>( \hat{\beta}_0 )</th>
<th>( \hat{\beta}_1 )</th>
<th>( \hat{\beta}_2 )</th>
<th>( \hat{\beta}_3 )</th>
<th>( \hat{\beta}_4 )</th>
<th>Variance ( (\sigma^2) )</th>
<th>F-test Ratio of variances ( (F_0) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>33182</td>
<td>11.270</td>
<td>-1.941</td>
<td></td>
<td></td>
<td>24.21</td>
<td>2.55</td>
</tr>
<tr>
<td>3</td>
<td>33173</td>
<td>24.220</td>
<td>-5.852</td>
<td>0.311</td>
<td></td>
<td>9.49</td>
<td>1.15</td>
</tr>
<tr>
<td>4</td>
<td>33178</td>
<td>12.630</td>
<td>0.632</td>
<td>-0.908</td>
<td>0.072</td>
<td>8.24</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 2. Polynomial fitting of the base station data
(a) 2\(^{nd}\) order (b) 3\(^{rd}\) order (c) 4\(^{th}\) order

In order to assess the performance of the proposed technique, the data corrected using the existing (conventional) method was smoothened using spline smooth filter of the Golden software (surfer), and then contoured. The same data set, corrected using the polynomial fitting technique was also spline smoothened and contoured using the same contour interval. The two plots (figure 3b and 3c respectively) have been compared with the plot of the same set of data (figure 3a) that has been
subjected to the same filtering and contouring processes but without application of the diurnal correction. The intension here was to assess the performance of the proposed method qualitatively. For example, the contour plot of the uncorrected data displays some level of noise at certain locations

![Contour plots](image)

**Fig. 3.** Effect of diurnal correction on the detailed magnetic data  
(a) Before correction (b) corrected using conventional method  
(c) corrected using polynomial fitting method

(labelled 1 to 5) which, though appear to be due to heading error (particularly location 5) can, most probably, be attributed to effect of diurnal variation. Such spurious events are still present in the field data corrected using the conventional method (figure 3b). On the other hand, the same set of data corrected using the proposed technique (figure 3c) displays adequate suppression of the diurnal effects.
4. Discussions

One of the main tasks in the application of the least-squares method in diurnal correction is getting the appropriate model for the base station data. Wrong choice of model for a set of data results wrong base station field leading to wrong calculation of the diurnal correction. For this reason, the base station data were carefully modelled. That is, the fitting procedure did not stop at obtaining the parameters of the model (table 1), and how well the computed models fit the data (figure 1), but included the statistical measure of the goodness-of-fit (table 1).

In order to assess how efficiently the proposed technique can be applied to archaeological magnetic data, or any other detailed magnetic survey, the result obtained was compared with that of the conventional method and also with the same set of data that was not subjected to diurnal correction. It is obvious that what appeared like heading errors (labelled 1 to 5) in figure 3a were actually part of the magnetic noise introduced into the total field data by diurnal variation. Such noise level can mask the useful information contained in the magnetic data, or introduce some fictitious anomalies into the processed data if not corrected.

5. Conclusion

The fitting of polynomial models to the base station data using the least-squares technique is an improvement on the existing method of magnetic data reduction. Without assuming linear variation in the geomagnetic field, the method suppresses the spurious signal superimposed on the total magnetic field data, and also produces fields of higher resolutions than conventional method given the same set of data. It also has the advantage of lower cost over the method that uses the base station magnetometer. Nevertheless, care should be taken not to fit higher order polynomials than is absolutely necessary due to inaccuracy and considerable error that can be introduced into the parameter estimate during matrix inversion calculations used in the least-squares technique.

References


